APPLIED GAS DYNAMICS

ETHIRAJAN RATHAKRISHNAN

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Contents

Preface *xv*
Author Biography *xvii*
About the Companion Website *xix*

1 Basic Facts *1*
1.1 Definition of Gas Dynamics *1*
1.2 Introduction *1*
1.3 Compressibility *2*
1.3.1 Limiting Conditions for Compressibility *3*
1.4 Supersonic Flow – What Is it? *4*
1.5 Speed of Sound *5*
1.6 Temperature Rise *7*
1.7 Mach Angle *8*
1.7.1 Small Disturbance *10*
1.7.2 Finite Disturbance *10*
1.8 Thermodynamics of Fluid Flow *11*
1.9 First Law of Thermodynamics (Energy Equation) *11*
1.9.1 Energy Equation for an Open System *12*
1.9.2 Adiabatic Flow Process *14*
1.10 The Second Law of Thermodynamics (Entropy Equation) *15*
1.11 Thermal and Calorical Properties *16*
1.11.1 Thermally Perfect Gas *16*
1.12 The Perfect Gas *17*
1.12.1 Entropy Calculation *18*
1.12.2 Isentropic Relations *20*
1.12.3 Limitations on Air as a Perfect Gas *25*
1.13 Wave Propagation *26*
1.14 Velocity of Sound *26*
1.15 Subsonic and Supersonic Flows *27*
1.16 Similarity Parameters *28*
1.17 Continuum Hypothesis *28*
1.18 Compressible Flow Regimes *30*
1.19 Summary *31*
Exercise Problems *34*

2 Steady One-Dimensional Flow *43*
2.1 Introduction *43*

2.2 Fundamental Equations *43*
2.3 Discharge from a Reservoir *45*
2.3.1 Mass Flow Rate per Unit Area *47*
2.3.2 Critical Values *51*
2.4 Streamtube Area–Velocity Relation *54*
2.5 de Laval Nozzle *57*
2.5.1 Mass Flow Relation in Terms of Mach Number *65*
2.5.2 Maximum Mass Flow Rate per Unit Area *65*
2.6 Supersonic Flow Generation *66*
2.6.1 Nozzles *68*
2.6.2 Physics of the Nozzle Flow Process *69*
2.7 Performance of Actual Nozzles *71*
2.7.1 Nozzle Efficiency *71*
2.7.2 Nozzle Discharge Coefficient *73*
2.8 Diffusers *75*
2.8.1 Special Features of Supersonic Diffusers *77*
2.8.2 Supersonic Wind Tunnel Diffusers *78*
2.8.3 Supersonic Inlets *81*
2.8.4 Fixed-Geometry Inlet *82*
2.8.5 Variable-Geometry Inlet *83*
2.8.6 Diffuser Efficiency *84*
2.9 Dynamic Head Measurement in Compressible Flow *88*
2.9.1 Compressibility Correction to Dynamic Pressure *91*
2.10 Pressure Coefficient *95*
2.11 Summary *97*
Exercise Problems *99*

3 Normal Shock Waves *113*
3.1 Introduction *113*
3.2 Equations of Motion for a Normal Shock Wave *113*
3.3 The Normal Shock Relations for a Perfect Gas *115*
3.4 Change of Stagnation or Total Pressure Across a Shock *118*
3.5 Hugoniot Equation *121*
3.5.1 Moving Shocks *123*
3.6 The Propagating Shock Wave *123*
3.6.1 Weak Shock *128*
3.6.2 Strong Shock *130*
3.7 Reflected Shock Wave *133*
3.8 Centered Expansion Wave *138*
3.9 Shock Tube *139*
3.9.1 Shock Tube Applications *142*
3.10 Summary *145*
Exercise Problems *148*

4 Oblique Shock and Expansion Waves *155*
4.1 Introduction *155*
4.2 Oblique Shock Relations *156*
4.3 Relation Between β and θ *158*
4.4 Shock Polar *160*

4.5 Supersonic Flow Over a Wedge *162*
4.6 Weak Oblique Shocks *165*
4.7 Supersonic Compression *167*
4.8 Supersonic Expansion by Turning *169*
4.9 The Prandtl–Meyer Expansion *170*
4.9.1 Velocity Components V_r and V_ϕ *172*
4.9.2 The Prandtl–Meyer Function *175*
4.9.3 Compression *177*
4.10 Simple and Nonsimple Regions *178*
4.11 Reflection and Intersection of Shocks and Expansion Waves *178*
4.11.1 Intersection of Shocks of the Same Family *181*
4.11.2 Wave Reflection from a Free Boundary *183*
4.12 Detached Shocks *189*
4.13 Mach Reflection *191*
4.14 Shock-Expansion Theory *197*
4.15 Thin Airfoil Theory *202*
4.15.1 Application of Thin Aerofoil Theory *203*
4.16 Summary *210*
Exercise Problems *212*

5 Compressible Flow Equations *221*
5.1 Introduction *221*
5.2 Crocco's Theorem *221*
5.2.1 Basic Solutions of Laplace's Equation *224*
5.3 General Potential Equation for Three-Dimensional Flow *225*
5.4 Linearization of the Potential Equation *226*
5.4.1 Small Perturbation Theory *227*
5.5 Potential Equation for Bodies of Revolution *229*
5.5.1 Conclusions *230*
5.5.2 Solution of Nonlinear Potential Equation *231*
5.6 Boundary Conditions *231*
5.6.1 Bodies of Revolution *232*
5.7 Pressure Coefficient *233*
5.7.1 Bodies of Revolution *234*
5.8 Summary *234*
Exercise Problems *237*

6 Similarity Rule *239*
6.1 Introduction *239*
6.2 Two-Dimensional Flow: The Prandtl–Glauert Rule for Subsonic Flow *239*
6.2.1 Prandtl–Glauert Transformations *239*
6.2.2 The Direct Problem (Version I) *241*
6.2.3 The Indirect Problem (Case of Equal Potentials): P–G Transformation (Version II) *243*
6.2.4 Streamline Analogy (Version III): Gothert's Rule *244*
6.3 Prandtl–Glauert Rule for Supersonic Flow: Versions I and II *245*
6.3.1 Subsonic Flow *246*
6.3.2 Supersonic Flow *246*
6.3.2.1 Analogy Version I *246*

6.3.2.2 Analogy Version II *247*
6.3.2.3 Analogy Version III: Gothert's Rule *247*
6.4 The von Karman Rule for Transonic Flow *248*
6.4.1 Use of the von Karman Rule *249*
6.5 Hypersonic Similarity *250*
6.6 Three-Dimensional Flow: Gothert's Rule *252*
6.6.1 General Similarity Rule *252*
6.6.2 Gothert's Rule *254*
6.6.3 Application to Wings of Finite Span *255*
6.6.3.1 Planform *255*
6.6.3.2 Profile *255*
6.6.4 Application to Bodies of Revolution and Fuselages *255*
6.6.4.1 Comparison of Two-Dimensional Symmetric Body and Axially Symmetric Body *256*
6.6.5 The Prandtl–Glauert Rule *257*
6.6.5.1 General Considerations *258*
6.6.5.2 P–G Rule for Two-Dimensional Flow, Using Eqs. (6.46) and (6.47) *258*
6.6.5.3 Application to Wings *259*
6.6.5.4 Application to Wings of Finite Span *260*
6.6.5.5 Application to Bodies of Revolution *261*
6.6.6 The von Karman Rule for Transonic Flow *261*
6.6.6.1 Application to Wings *261*
6.6.6.2 Application to Bodies of Revolution *261*
6.7 Critical Mach Number *261*
6.7.1 Calculation of M_∞^* *264*
6.8 Summary *266*
Exercise Problems *269*

7 Two-Dimensional Compressible Flows *271*
7.1 Introduction *271*
7.2 General Linear Solution for Supersonic Flow *271*
7.2.1 Existence of Characteristics in a Physical Problem *273*
7.2.2 Equation for the Streamlines from Kinematic Flow Condition *274*
7.3 Flow over a Wave-Shaped Wall *276*
7.3.1 Incompressible Flow *276*
7.3.2 Compressible Subsonic Flow *277*
7.3.3 Supersonic Flow *278*
7.3.4 Pressure Coefficient *278*
7.4 Summary *280*
Exercise Problems *280*

8 Flow with Friction and Heat Transfer *283*
8.1 Introduction *283*
8.2 Flow in Constant Area Duct with Friction *283*
8.2.1 The Fanno Line *284*
8.3 Adiabatic, Constant-Area Flow of a Perfect Gas *285*
8.3.1 Definition of Friction Coefficient *286*
8.3.2 Effects of Wall Friction on Fluid Properties *287*
8.3.3 Second Law of Thermodynamics *288*

8.3.4 Working Relations *289*
8.4 Flow with Heating or Cooling in Ducts *294*
8.4.1 Governing Equations *294*
8.4.2 Simple-Heating Relations for a Perfect Gas *295*
8.5 Summary *300*
Exercise Problems *303*

9 Method of Characteristics *309*
9.1 Introduction *309*
9.2 The Concepts of Characteristics *309*
9.3 The Compatibility Relation *310*
9.4 The Numerical Computational Method *312*
9.4.1 Solid and Free Boundary Points *313*
9.4.2 Sources of Error *316*
9.4.3 Axisymmetric Flow *316*
9.4.4 Nonisentropic Flow *317*
9.5 Theorems for Two-Dimensional Flow *318*
9.6 Numerical Computation with Weak Finite Waves *320*
9.6.1 Reflection of Waves *320*
9.7 Design of Supersonic Nozzle *323*
9.7.1 Contour Design Details *324*
9.8 Summary *328*

10 Measurements in Compressible Flow *329*
10.1 Introduction *329*
10.2 Pressure Measurements *329*
10.2.1 Liquid Manometers *329*
10.2.2 Measuring Principle of Manometers *330*
10.2.3 Dial-Type Pressure Gauges *332*
10.2.4 Pressure Transducers *333*
10.3 Temperature Measurements *335*
10.4 Velocity and Direction *338*
10.5 Density Problems *339*
10.6 Compressible Flow Visualization *339*
10.6.1 Supersonic Flows *340*
10.7 Interferometer *341*
10.7.1 Formation of Interference Patterns *341*
10.7.2 Quantitative Evaluation *342*
10.7.3 Fringe-Displacement Method *344*
10.8 Schlieren System *344*
10.8.1 Range and Sensitivity of the Schlieren System *347*
10.8.2 Optical Components Quality Requirements *347*
10.8.2.1 Schlieren Mirrors *347*
10.8.2.2 Light Source *348*
10.8.2.3 Condenser Lens *348*
10.8.2.4 Focusing Lens *348*
10.8.2.5 Knife-Edge *348*
10.8.2.6 Color Schlieren *348*
10.8.2.7 Short-Duration Light Source *348*

10.8.3 Sensitivity of the Schlieren Method for Shock and Expansion Studies *350*
10.9 Shadowgraph *352*
10.9.1 Comparison of the Schlieren and Shadowgraph Methods *353*
10.10 Wind Tunnels *354*
10.10.1 High-Speed Wind Tunnels *354*
10.10.2 Blowdown Type Wind Tunnels *354*
10.10.2.1 Advantages *355*
10.10.2.2 Disadvantages *355*
10.10.3 Induction Type Tunnels *355*
10.10.3.1 Advantages *356*
10.10.3.2 Disadvantages *356*
10.10.4 Continuous Supersonic Wind Tunnels *356*
10.10.5 Losses in Supersonic Tunnels *357*
10.10.6 Supersonic Wind Tunnel Diffusers *358*
10.10.6.1 Polytropic Efficiency *358*
10.10.6.2 Isentropic Efficiency *359*
10.10.7 Effects of Second Throat *360*
10.10.8 Compressor Tunnel Matching *362*
10.10.9 The Mass Flow Rate *365*
10.10.10 Blowdown Tunnel Operation *369*
10.10.10.1 Reynolds Number Control *370*
10.10.11 Optimum Conditions *372*
10.10.12 Running Time of Blowdown Wind Tunnels *373*
10.11 Hypersonic Tunnels *375*
10.11.1 Hypersonic Nozzle *377*
10.12 Instrumentation and Calibration of Wind Tunnels *380*
10.12.1 Calibration of Supersonic Wind Tunnels *380*
10.12.2 Calibration *381*
10.12.3 Mach Number Determination *381*
10.12.4 Pitot Pressure Measurement *382*
10.12.5 Static Pressure Measurement *382*
10.12.6 Determination of Flow Angularity *383*
10.12.7 Determination of Turbulence Level *383*
10.12.8 Determination of Test-Section Noise *384*
10.12.9 Use of Calibration Results *384*
10.12.10 Starting of Supersonic Tunnels *384*
10.12.11 Starting Loads *385*
10.12.12 Reynolds Number Effects *385*
10.12.13 Model Mounting-Sting Effects *385*
10.13 Calibration and Use of Hypersonic Tunnels *386*
10.13.1 Calibration of Hypersonic Tunnels *386*
10.13.2 Mach Number Determination *386*
10.13.3 Determination of Flow Angularity *388*
10.13.4 Determination of Turbulence Level *388*
10.13.4.1 Blockage Tests *388*
10.13.4.2 Starting Loads *388*
10.13.5 Reynolds Number Effects *389*
10.13.6 Force Measurements *389*
10.14 Flow Visualization *390*

10.15 Summary *390*
Exercise Problems *393*

11 Ramjet *395*
11.1 Introduction *395*
11.2 The Ideal Ramjet *396*
11.3 Aerodynamic Losses *401*
11.4 Aerothermodynamics of Engine Components *404*
11.4.1 Engine Inlets *404*
11.4.1.1 Subsonic Inlets *404*
11.5 Flow Through Inlets *405*
11.5.1 Inlet Flow Process *406*
11.5.2 Boundary Layer Separation *406*
11.5.3 Flow Over the Inlet *406*
11.6 Performance of Actual Intakes *410*
11.6.1 Isentropic Efficiency *410*
11.6.2 Stagnation Pressure Ratio *411*
11.6.3 Supersonic Inlets *411*
11.6.4 Supersonic Diffusers *412*
11.6.5 Starting Problem *413*
11.7 Shock–Boundary Layer Interaction *418*
11.8 Oblique Shock Wave Incident on Flat Plate *419*
11.9 Normal Shocks in Ducts *420*
11.10 External Supersonic Compression *422*
11.11 Two-Shock Intakes *423*
11.12 Multi-Shock Intakes *427*
11.13 Isentropic Compression *429*
11.14 Limits of External Compression *431*
11.15 External Shock Attachment *433*
11.16 Internal Shock Attachment *433*
11.17 Pressure Loss *434*
11.18 Supersonic Combustion *442*
11.19 Summary *444*
Exercise Problems *447*

12 Jets *451*
12.1 Introduction *451*
12.1.1 Subsonic Jets *453*
12.2 Mathematical Treatment of Jet Profiles *454*
12.3 Theory of Turbulent Jets *455*
12.3.1 Mean Velocity and Mean Temperature *456*
12.3.2 Turbulence Characteristics of Free Jets *457*
12.3.3 Mixing Length *458*
12.4 Experimental Methods for Studying Jets and the Techniques Used for Analysis *461*
12.4.1 Pressure Measurement *462*
12.4.1.1 Precautions Observed *463*
12.5 Expansion Levels of Jets *464*
12.5.1 Overexpanded Jets *464*

12.5.2 Correctly Expanded Jets *467*
12.5.3 Underexpanded Jets *469*
12.6 Control of Jets *471*
12.6.1 Classification of Control Methods *473*
12.6.1.1 Active Control *473*
12.6.1.2 Passive Control *473*
12.6.2 Role of Shear Layer in Flow Control *474*
12.6.2.1 Large-Scale Structure in Subsonic Shear Layers *474*
12.6.2.2 Role of Large-Scale Structures in Subsonic Mixing Enhancement *475*
12.6.3 Supersonic Shear Layers *475*
12.6.4 Use of Tabs for Jet Control *477*
12.6.5 Evaluation of the Effectiveness of Some Specific Passive Controls *481*
12.6.5.1 Jet Control with the Limiting Tab *481*
12.6.5.2 Centerline Decay *482*
12.6.5.3 Flow Development *487*
12.6.5.4 Acoustic Characteristics of a Mach 1.6 Jet *490*
12.6.5.5 Acoustic Characteristics of a Mach 1.79 Jet *496*
12.6.5.6 Acoustic Characteristics of a Mach 2 Jet *496*
12.6.5.7 Use of Corrugated Tabs *497*
12.6.5.8 Pressure Profiles *504*
12.6.5.9 Optical Flow Visualization *511*
12.6.5.10 Water Flow Visualization *517*
12.6.6 Grooves and Cutouts *519*
12.7 Noncircular Jets and Shifted Tabs *519*
12.7.1 Jet Control with Tabs *523*
12.7.2 Shifted Tabs *527*
12.7.3 Ventilated Triangular Tabs *532*
12.7.4 Tab Edge Effect *535*
12.8 Summary *541*

Appendix A *547*
References *619*

Index *625*

Preface

The first edition of this book, developed to serve as the text for a course on gas dynamics at the introductory level for undergraduates and an advanced level at the graduate level was well received all over the world, because of its completeness and proper balance between the theoretical aspects and application of this science.

Over the years, the feedback received from the faculty and students made the author realize the need for adding solved examples and exercise problems at the end of the chapter on ramjets.

Some faculty suggested adding more examples at appropriate locations in various chapters and adding exercise problems on moving waves and similarity would make the book more effective for practicing the theory presented in the book.

Considering the feedback from faculty and students, the following material has been added to this edition.

- A number of new examples to different chapters of the book.
- A section on critical Mach number in Chapter 6.
- Examples using the theory of ramjet and a list of exercise problems along with answers in Chapter 11.
- A subsection highlighting new developments in the field of noncircular jets and the control of jets with shifted tab, which keeps the nozzle exit clean so that the jet will enjoy full momentum thrust potential in Chapter 12.

I would like to thank the faculty and students all over the world for adopting this book for their courses. I thank my doctoral student Maruthupandiyan and master's student Kaustubh Hirve for checking the material added in this edition and the Solutions Manual.

For instructors, a companion Solutions Manual that contains typed solutions to all the end-of-chapter problems and lecture slides for the complete book are available from the publisher.

Ethirajan Rathakrishnan

Author Biography

Ethirajan Rathakrishnan is professor of Aerospace Engineering at the Indian Institute of Technology Kanpur. He is well known internationally for his research in the area of high-speed jets. The limit for the passive control of jets, called the *Rathakrishnan Limit*, is his contribution to the field of jet research, and the concept of *breathing blunt nose (BBN)*, which reduces the positive pressure at the nose and increases the low pressure at the base simultaneously, is his contribution to drag reduction at hypersonic speeds. Positioning the twin-vortex Reynolds number at around 5000, by changing the geometry from cylinder, for which the maximum limit for the Reynolds number for positioning the twin-vortex was found to be around 160, by von Karman, to flat plate, is his addition to vortex flow theory. He has published a large number of research articles in many highly regarded international journals. He is a Fellow of many professional societies, including the Royal Aeronautical Society. Professor Rathakrishnan serves as the editor-in-chief of the *International Review of Aerospace Engineering* (IREASE) and *International Review of Mechanical Engineering* (IREME) journals. He has authored 12 other books: *Gas Dynamics*, 6th ed. (PHI Learning, New Delhi, 2017); *Fundamentals of Engineering Thermodynamics*, 2nd ed. (PHI Learning, New Delhi, 2005); *Fluid Mechanics: An Introduction*, 3rd ed. (PHI Learning, New Delhi, 2012); *Gas Tables*, 3rd ed. (Universities Press, Hyderabad, India, 2012); *Theory of Compressible Flows* (Maruzen Co., Ltd., Tokyo, 2008); *Gas Dynamics Work Book*, 2nd ed. (Praise Worthy Prize, Napoli, 2013); *Elements of Heat Transfer* (CRC Press, Boca Raton, FL, 2012); *Theoretical Aerodynamics* (Wiley, Hoboken, NJ, 2013); *High Enthalpy Gas Dynamics* (Wiley, Hoboken, NJ, 2015); *Dynamique Des Gaz* (Praise Worthy Prize, Napoli, Italy, 2015); *Instrumentation, Measurements and Experiments in Fluids*, 2nd ed. (CRC Press, Boca Raton, FL, 2017); and *Helicopter Aerodynamics* (PHI Learning, New Delhi, 2019).

At least two trees are planted for each one used for paper production.

Ethirajan Rathakrishnan

About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/gasdyn

The website includes:

- Lecture slides for the complete book
- Solutions Manual containing detailed solutions for the problems listed at the end of different chapters

1

Basic Facts

1.1 Definition of Gas Dynamics

Gas dynamics is the science of fluid flow in which both density and temperature changes become significant. Taking 5% change in temperature as significant, it can be stated that, at standard sea level, Mach number 0.5 is the lower limit of gas dynamics. Thus, gas dynamics is the science of flow fields with speeds of Mach 0.5 and above. Therefore, gas dynamic regimes consist of both subsonic and supersonic Mach numbers. Further, when the flow is supersonic, any change of flow property or direction is caused by waves. These waves are isentropic and nonisentropic compression waves (shock waves), expansion waves, and Mach waves. Among these, the compression and expansion waves can cause finite changes but the flow property changes caused by a Mach wave are insignificant. The essence of gas dynamics is that, when the flow speed is supersonic, the entire flow field is dominated by Mach waves, expansion waves, and shock waves. It is through these waves that the change of flow properties, from one state to another, takes place.

1.2 Introduction

Compressible flow is the science of fluid flow where the density change associated with pressure change is significant. *Fluid mechanics* is the science of fluid flow in which the temperature changes associated with the flow are insignificant. Fluid mechanics is essentially the science of *isenthalpic* flows, and thus the main equations governing a fluid dynamic stream are only the continuity and momentum equations plus the second law of thermodynamics. The energy equation is passive as far as fluid dynamic streams are concerned. At standard sea level conditions, considering less than 5% change in temperature as insignificant, flow with a Mach number of less than 0.5 can be termed a *fluid mechanic stream*. A fluid mechanic stream may be compressible or incompressible. For an incompressible flow, both temperature and density changes are insignificant. For a compressible flow, the temperature change may be insignificant but density change is finite.

However, in many engineering applications, such as the design of airplanes, missiles, and launch vehicles, the flow Mach numbers associated are more than 0.5. Hence both temperature and density changes associated with the flow become significant. The study of such flows where both density and temperature changes associated with pressure change become appreciable is called *gas dynamics*. In other words, gas dynamics is the science of fluid flow in which both density and temperature changes are significant. The essence of gas dynamics is that the entire flow field is dominated by Mach waves, expansion waves, and shock waves, when the flow speed is supersonic. It is through these waves that the change of flow properties

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from one state to another takes place. In the theory of gas dynamics, a change of state in flow properties is achieved by three means: (i) with area change, treating the fluid as inviscid and passage to be frictionless; (ii) with friction, treating the heat transfer between the surroundings and the system to be negligible; and (iii) with heat transfer, assuming the fluid to be inviscid. These three types of flows are called *isentropic flow*, *frictional* or *Fanno type flow*, and *Rayleigh type flow*, respectively.

All problems in gas dynamics can be classified under the three flow processes described above, while, of course, bearing in mind the previously stated assumptions. Although it is impossible in practice to have a flow process which is purely isentropic or Fanno type or Rayleigh type, these assumptions are justified, since the results obtained with these treatments prove to be accurate enough for most practical problems in gas dynamics.

1.3 Compressibility

Fluids such as water are incompressible under normal conditions. But under conditions of high pressure (e.g. 1000 atm) they are compressible. The change in volume is the characteristic feature of a compressible medium under static conditions. Under dynamic conditions, that is when the medium is moving, the characteristic feature for incompressible and compressible flow situations are: the volume flow rate, $\dot{Q} = AV =$ constant at any cross-section of a streamtube for incompressible flow, and the mass flow rate, $\dot{m} = \rho AV =$ constant at any cross-section of a streamtube for compressible flow. In these relations, A is the cross-sectional area of the streamtube and V and ρ are, respectively, the velocity and density of the fluid at that cross-section (Figure 1.1).

In general, the flow of an incompressible medium is called *incompressible flow* and that of a compressible medium is called *compressible flow*. Though this statement is true for incompressible media under normal conditions of pressure and temperature, for compressible media, like gases, it has to be modified.

As long as a gas flows at a sufficiently low speed from one cross-section of a passage to another the change in volume (or density) can be neglected and, therefore, the flow can be treated as incompressible. Although the fluid is compressible, this property may be neglected when the flow is taking place at low speeds. In other words, although there is some density change associated with every physical flow, it is often possible (for low-speed flows) to neglect it and idealize the flow as incompressible. This approximation is applicable to many practical flow situations, such as low-speed flow around an airplane and flow through a vacuum cleaner.

From the above discussion it is clear that compressibility is the phenomenon by virtue of which the flow changes its density with changes in speed. Now, we may ask, what are the precise conditions under which density changes must be considered? We will try to answer this question now.

A quantitative measure of compressibility is the volume modulus of elasticity E, defined as

$$E = -\frac{\Delta p}{\Delta \mathbf{V}/\mathbf{V}_i} \tag{1.1}$$

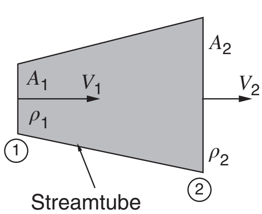


Figure 1.1 Elemental streamtube.

where Δp is the change in static pressure, $\Delta \mathbf{V}$ is the change in volume, and $\mathbf{V}_i$ is the initial volume. For ideal gases, the equation of state is

$$p\mathbf{V} = RT$$

For isothermal flows, this reduces to

$$p\mathbf{V} = p_i\mathbf{V}_i = \text{constant}$$

where p_i is the initial pressure.

The above equation may be written as

$$(p_i + \Delta p)(\mathbf{V}_i + \Delta \mathbf{V}) = p_i\mathbf{V}_i$$

Expanding this equation, and neglecting the second-order terms, we get

$$\Delta p\mathbf{V}_i + \Delta \mathbf{V} p_i = 0$$

Therefore,

$$\Delta p = -p_i \frac{\Delta \mathbf{V}}{\mathbf{V}_i} \tag{1.2}$$

For gases, from Eqs. (1.1) and (1.2), we get

$$E = p_i \tag{1.3}$$

Hence, by Eq. (1.2), the compressibility may be defined as the volume modulus of the pressure.

1.3.1 Limiting Conditions for Compressibility

By mass conservation, we have $\dot{m} = \rho V = \text{constant}$, where $\dot{m}$ is mass flow rate per unit area, V is the flow velocity, and ρ is the corresponding density. This can also be written as

$$(V_i + \Delta V)(\rho_i + \Delta \rho) = \rho_i\, V_i$$

Considering only first-order terms, this simplifies to

$$\frac{\Delta \rho}{\rho_i} = -\frac{\Delta V}{V_i}$$

Substituting this into Eq. (1.1) and noting that $V = \mathbf{V}$ for unit area per unit time in the present case, we get

$$\Delta p = E\frac{\Delta \rho}{\rho_i} \tag{1.4}$$

From Eq. (1.4), it can be seen that the compressibility may also be defined as the *density modulus* of the pressure.

For incompressible flows, by Bernoulli's equation, we have

$$p + \frac{1}{2}\rho\, V^2 = \text{constant} = p_{\text{stag}}$$

where the subscript “stag” refers to *stagnation condition*. The above equation may also be written as

$$p_{\text{stag}} - p = \Delta p = \frac{1}{2}\rho\, V^2$$

that is the change of pressure from stagnation to static states is equal to $\frac{1}{2}\rho V^2$. Using Eq. (1.4) in the above relation, we obtain

$$\frac{\Delta p}{E} = \frac{\Delta \rho}{\rho_i} = \frac{\rho_i V_i^2}{2E} = \frac{q_i}{E} \tag{1.5}$$

where $q_i = \frac{1}{2}\rho_i V_i^2$ is the dynamic pressure. Equation (1.5) relates the density change to the flow speed.

The compressibility effects can be neglected if the density changes are very small, that is if

$$\frac{\Delta\rho}{\rho_i} \ll 1$$

From Eq. (1.5) it is seen that for neglecting compressibility

$$q/E \ll 1$$

For gases, the speed of sound a may be expressed in terms of pressure and density changes as (see Eq. (1.11))

$$a^2 = \frac{\Delta p}{\Delta\rho}$$

Using Eq. (1.4) in the above relation, we get

$$a^2 = \frac{E}{\rho_i}$$

With this, Eq. (1.5) reduces to

$$\frac{\Delta\rho}{\rho_i} = \frac{\rho_i}{2}\frac{V_i^2}{E} = \frac{1}{2}\left(\frac{V}{a}\right)^2 \tag{1.6}$$

The ratio V/a is called the Mach number M. Therefore, the condition of incompressibility for gases becomes

$$M^2/2 \ll 1$$

Thus, the criterion determining the effect of compressibility for gases is that the magnitude of the Mach number M should be negligibly small. Indeed, mathematics would stipulate this limit as $M \to 0$. But Mach number zero corresponds to stagnation state. Therefore, in engineering sciences flows with very small Mach numbers are treated as incompressible. To have a quantification of this limiting value of the Mach number to treat a flow as incompressible, a Mach number corresponding to a 5% change in flow density is usually taken as the limit.

It is widely accepted that compressibility can be neglected when

$$\frac{\Delta\rho}{\rho_i} \leq 0.05 \text{ or } 5\%$$

that is when $M \leq 0.3$. In other words, the flow may be treated as incompressible when $V \leq 100\,\text{m s}^{-1}$, that is when $V \leq 360\,\text{kmph}$ under standard sea level conditions. The above values of M and V are widely accepted values and they may be re-fixed at different levels, depending upon the flow situation and the degree of accuracy desired.

1.4 Supersonic Flow – What Is it?

The Mach number M is defined as the ratio of the local flow speed V to the local speed of sound a

$$\boxed{M = \frac{V}{a}} \tag{1.7}$$

Thus, M is a dimensionless quantity. In general, both V and a are functions of position and time. Therefore, the Mach number is not just the flow speed made nondimensional by dividing by a constant. In other words, the flow Mach number is the ratio of V to a and this relation should not be viewed as M proportional to V, or inversely proportional to a, in isolation. That is, we cannot write $M \propto V$ or $M \propto 1/a$ in isolation. However, it is almost always true that M increases monotonically with V.

A flow with a Mach number greater than unity is termed *supersonic flow*. In a supersonic flow $V > a$ and the flow upstream of a given point remains unaffected by changes in conditions at that point.

1.5 Speed of Sound

Sound waves are infinitely small pressure disturbances. The speed with which sound propagates in a medium is called the *speed of sound* and is denoted by a. If an infinitesimal disturbance is created by the piston, as shown in Figure 1.2, the wave propagates through the gas at the velocity of sound relative to the gas into which the disturbance is moving. Let the stationary gas at pressure p_i and density ρ_i in the pipe be set in motion by moving the piston. The infinitesimal pressure wave created by the piston movement travels with speed a, leaving the medium behind it at pressure p_1 and density ρ_1 to move with velocity V.

As a result of compression created by the piston, the pressure and density next to the piston are infinitesimally greater than the pressure and density of the gas at rest ahead of the wave. Therefore,

$$\Delta p = p_1 - p_i, \quad \Delta \rho = \rho_1 - \rho_i$$

are small.

Choose a control volume of length b, as shown in Figure 1.2. Compression of volume Ab causes the density to rise from ρ_i to ρ_1 in time $t = b/a$. The mass flow into volume Ab is

$$\dot{m} = \rho_1 AV \tag{1.8}$$

For mass conservation, $\dot{m}$ must also be equal to the mass flow rate $A\, b(\rho_1 - \rho_i)/t$ through the control volume. Thus,

$$Ab(\rho_1 - \rho_i)/t = \rho_1 AV$$

or

$$a(\rho_1 - \rho_i) = \rho_1 V \tag{1.9}$$

because $b/t = a$.

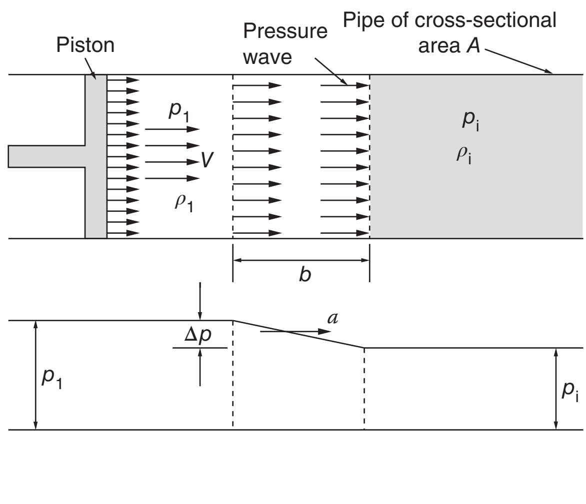


Figure 1.2 Propagation of pressure disturbance.

The compression wave caused by the piston motion travels and accelerates the gas from zero velocity to V. The acceleration is given by

$$\frac{V}{t} = V\frac{a}{b}$$

The mass in the control volume Ab is

$$m = Ab\overline{\rho}$$

where

$$\overline{\rho} = \frac{\rho_i + \rho_1}{2}$$

The force acting on the control volume is $F = A(p_1 - p_i)$. Therefore, by Newton's law,

$$A(p_1 - p_i) = m\left(V\frac{a}{b}\right)$$

$$A(p_1 - p_i) = (Ab\overline{\rho})\left(V\frac{a}{b}\right)$$

or

$$\overline{\rho}Va = p_1 - p_i \tag{1.10}$$

Because the disturbance is very weak, ρ_1 on the right-hand side of Eq. (1.9) may be replaced by $\overline{\rho}$ to result in

$$a(\rho_1 - \rho_i) = \overline{\rho}V$$

Using this relation, Eq. (1.10) can be written as

$$a^2 = \frac{p_1 - p_i}{\rho_1 - \rho_i} = \frac{\Delta p}{\Delta \rho}$$

In the limiting case of Δp and $\Delta \rho$ approaching zero, the above equation leads to

$$\boxed{a^2 = \frac{dp}{d\rho}} \tag{1.11}$$

This is the *Laplace equation* and is valid for any fluid.

The sound wave is an isentropic pressure wave, across which only infinitesimal changes in fluid properties occur. Further, the wave itself is extremely thin, and changes in properties occur very rapidly. The rapidity of the process rules out the possibility of any heat transfer between the system of fluid particles and its surrounding.

For very strong pressure waves, the traveling speed of a disturbance may be greater than that of sound. The pressure can be expressed as

$$p = p(\rho) \tag{1.12}$$

For an isentropic process of a gas,

$$\frac{p}{\rho^\gamma} = \text{constant}$$

where the isentropic index γ is the ratio of specific heats and is a constant for a perfect gas. Using the above relation in Eq. (1.11), we get

$$a^2 = \gamma p/\rho \tag{1.13}$$

For a perfect gas, by the state equation

$$p = \rho RT \tag{1.14}$$

where R is the gas constant and T the static temperature of the gas in absolute units.

Equations (1.13) and (1.14) together lead to the following expression for the speed of sound.

$$\boxed{a = \sqrt{\gamma RT}} \tag{1.15}$$

The perfect gas assumption is valid so long as the speed of the gas stream is not too high. However, at hypersonic speeds the assumption of a perfect gas is not valid and we must consider Eq. (1.11) to calculate the speed of sound.

1.6 Temperature Rise

For a perfect gas,

$$p = \rho RT, \ \ R = c_p - c_v$$

where c_p and c_v are specific heats at constant pressure and constant volume, respectively. Also, $\gamma = c_p/c_v$; therefore,

$$R = \frac{\gamma - 1}{\gamma} c_p \tag{1.16}$$

For an isentropic change of state, an equation not involving T can be written as

$$p/\rho^\gamma = \text{constant}$$

Now, between state 1 and any other state the relation between the pressures and densities can be written as

$$\left(\frac{p}{p_1}\right) = \left(\frac{\rho}{\rho_1}\right)^\gamma \tag{1.17}$$

Combining Eqs. (1.17) and (1.14), we get

$$\frac{T}{T_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma-1} = \left(\frac{p}{p_1}\right)^{(\gamma-1)/\gamma} \tag{1.18}$$

The above relations are very useful for gas dynamic studies. The temperature, density, and pressure ratios in Eq. (1.18) can be expressed in terms of the flow Mach number.

Let us examine the flow around a symmetrical body, as shown in Figure 1.3.

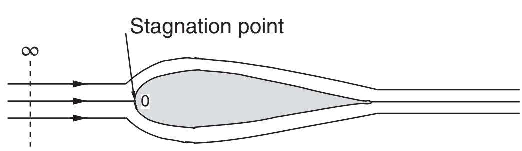
In a compressible medium, there will be a change in density and temperature at point 0. The temperature rise at the stagnation point can be obtained from the energy equation.

The energy equation for an isentropic flow is

$$h + \frac{V^2}{2} = \text{constant} \tag{1.19}$$

where h is the enthalpy.

Figure 1.3 Flow around a symmetrical body.



Equating the energy at far upstream, ∞, and the stagnation point 0, we get

$$h_\infty + \frac{V_\infty^2}{2} = h_0 + \frac{V_0^2}{2}$$

But $V_0 = 0$, thus

$$h_0 - h_\infty = \frac{V_\infty^2}{2}$$

For a perfect gas $h = c_p T$; therefore, from the above relation we obtain

$$c_p(T_0 - T_\infty) = \frac{V_\infty^2}{2}$$

that is

$$\Delta T = T_0 - T_\infty = \frac{V_\infty^2}{2c_p} \tag{1.20}$$

Combining Eqs. (1.15) and (1.16), we get

$$c_p = \frac{1}{\gamma - 1}\frac{a_\infty^2}{T_\infty}$$

Hence,

$$\Delta T = \frac{\gamma - 1}{2} T_\infty M_\infty^2 \tag{1.21}$$

that is

$$\boxed{T_0 = T_\infty\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)} \tag{1.22}$$

For air, $\gamma = 1.4$, and hence

$$T_0 = T_\infty(1 + 0.2M_\infty^2) \tag{1.23}$$

where T_0 is the temperature at the stagnation point on the body. It is also referred to as *total temperature*. For example, at the stagnation point 0 on the body shown in Figure 1.3, the flow will attain the stagnation temperature.

1.7 Mach Angle

The presence of a small disturbance is felt throughout the field by means of disturbance waves traveling at the local velocity of sound relative to the medium. Let us examine the propagation of pressure disturbances created by a moving object, shown in Figure 1.4. The propagation of disturbance waves created by an object moving with velocity $V = 0$, $V = a/2$, $V = a$, and $V > a$ is shown in Figures 1.4a–d, respectively. In a subsonic flow the disturbance waves reach a stationary observer before the source of disturbance could reach him, as shown in Figures 1.4a and b. But in supersonic flows it takes a considerable amount of time for an observer to perceive the pressure disturbance, after the source has passed. This is one of the fundamental differences between subsonic and supersonic flows. Therefore, in a subsonic flow the streamlines sense the presence of any obstacle in the flow field and adjust themselves well ahead of the obstacle and flow around it smoothly.

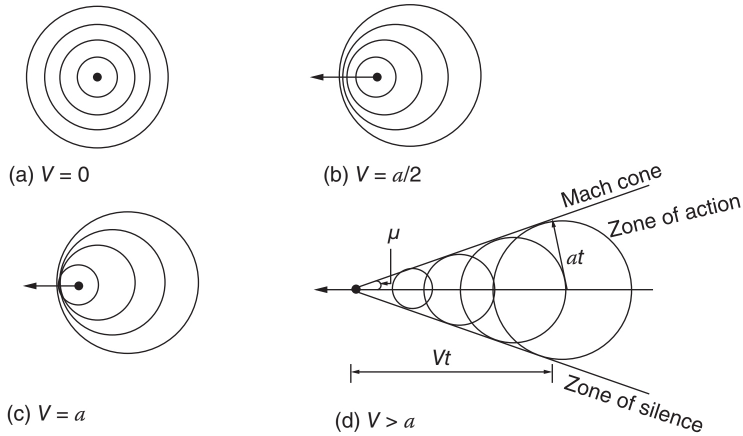


Figure 1.4 Propagation of disturbance waves.

But in a supersonic flow, the streamlines feel the obstacle only when they hit it. The obstacle acts as a source, and the streamlines deviate at the Mach cone, as shown in Figure 1.4d. Thus, in a supersonic flow, the disturbance due to an obstacle is sudden and the flow behind the obstacle has to change abruptly.

Flow around a wedge shown in Figures 1.5a and b illustrates the smooth and abrupt change in flow direction for subsonic and supersonic flow, respectively. For $M_\infty < 1$, the flow direction changes smoothly and the pressure decreases with acceleration. For $M_\infty > 1$, there is a sudden change in flow direction at the body, and the pressure increases downstream of the shock.

In Figure 1.4d, it is shown that for supersonic motion of an object there is a well-defined conical zone in the flow field with the object located at the nose of the cone, and the disturbance created by the moving object is confined only to the field included inside the cone. The flow field zone outside the cone does not even feel the disturbance. For this reason, von Karman termed the region inside the cone as the *zone of action*, and the region outside the cone as the *zone of silence*. The lines at which the pressure disturbance is concentrated and which generate the cone are called *Mach waves* or *Mach lines*. The angle between the Mach line and the direction of motion of the body is called the *Mach angle* μ. From Figure 1.4d, we have

$$\sin\mu = \frac{at}{Vt} = \frac{a}{V}$$

that is

$$\boxed{\sin\mu = \frac{1}{M}} \tag{1.24}$$

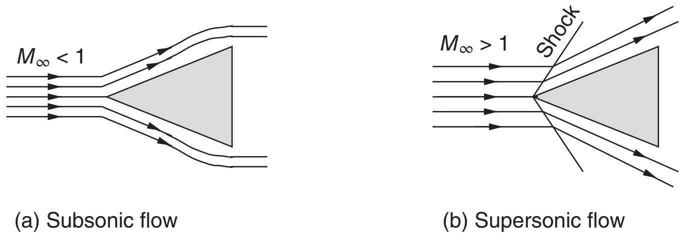


Figure 1.5 Flow around a wedge.

From the propagation of the disturbance waves shown in Figure 1.4, we can infer the following features of the flow regimes.

- When the medium is *incompressible* ($M = 0$, Figure 1.4a) or when the speed of the moving disturbance is negligibly small compared to the local sound speed, the pressure pulse created by the disturbance spreads uniformly in all directions.
- When the disturbance source moves with a *subsonic speed* ($M < 1$, Figure 1.4b), the pressure disturbance is felt in all directions and at all points in space (neglecting viscous dissipation), but the pressure pattern is no longer symmetrical.
- For *sonic velocity* ($M = 1$, Figure 1.4c) the pressure pulse is at the boundary between subsonic and supersonic flow and the wavefront is a plane.
- For *supersonic speeds* ($M > 1$, Figure 1.4d) the disturbance wave propagation phenomenon is totally different from those at subsonic speeds. All the pressure disturbances are included in a cone that has the disturbance source at its apex, and the effect of the disturbance is not felt upstream of the disturbance source.

1.7.1 Small Disturbance

When the apex angle of wedge δ is vanishingly small, the disturbances will be small, and we can consider these disturbance waves identical to sound pulses. In such a case, the deviation of streamlines will be small and there will be an infinitesimally small increase of pressure across the Mach cone, as shown in Figure 1.6.

1.7.2 Finite Disturbance

When the wedge angle δ is finite, the disturbances introduced are finite, then the wave is not called a Mach wave but a *shock* or shock wave (see Figure 1.7). The angle of shock β is always smaller than the Mach angle. The deviation of the streamlines is finite and the pressure increase across a shock wave is finite.

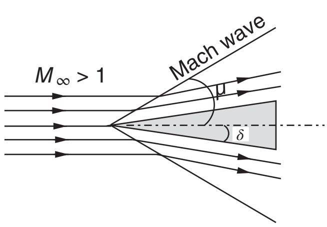


Figure 1.6 Mach cone.

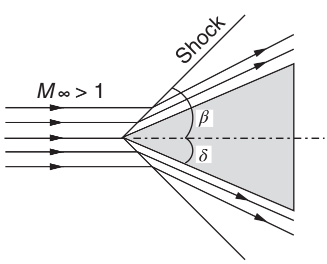


Figure 1.7 Shock wave.

1.8 Thermodynamics of Fluid Flow

Entropy and temperature are the two fundamental concepts of thermodynamics. Unlike low-speed or incompressible flows, the energy change associated with a compressible flow is substantial enough to strongly interact with other properties of the flow. Hence, the energy concept plays an important role in the study of compressible flows. In other words, the study of thermodynamics which deals with energy (and entropy) is an essential component in the study of compressible flow.

The following are the broad divisions of fluid flow based on thermodynamic considerations. (i) Fluid mechanics of perfect fluids – fluids without viscosity and heat (transfer) conductivity – is an extension of equilibrium thermodynamics to moving fluids. The kinetic energy of the fluid has to be considered in addition to the internal energy which the fluid possesses, even when at rest. (ii) Fluid mechanics of real fluids (those that go beyond the scope of classical thermodynamics). The transport processes of momentum and heat (energy) are of primary interest here. But, even though thermodynamics is not fully and directly applicable to all phases of real fluid flow, it is often extremely helpful in relating the initial and final conditions.

For low-speed flow problems, thermodynamic considerations are not needed, because the heat content of the fluid flow is so large compared to the kinetic energy of the flow that the temperature remains nearly constant even if the whole of the kinetic energy is transformed into heat. In other words, the difference between the static and stagnation temperatures is not significant in low-speed flows. But in high-speed flows, the kinetic energy content of the fluid can be so large compared to its heat content that the difference between the static and stagnation temperature can become substantial. Hence, emphasis on the thermodynamic concepts assumes importance in high-speed flow analysis.

1.9 First Law of Thermodynamics (Energy Equation)

Consider a closed system, consisting of a certain amount of gas at rest, across whose boundaries no transfer of mass is possible. Let δQ be an incremental amount of heat added to the system across the boundary (by thermal conduction or by direct radiation). Also, let δW denote the work done on the system by the surroundings (or by the system on the surroundings). The sign convention is positive when the work is done by the system and negative when the work is done on the system. Owing to the molecular motion of the gas, the system has an internal energy U. The first law of thermodynamics states that the *heat added minus work done by the system is equal to the change in the internal energy of the system:*

$$\boxed{\delta Q - \delta W = dU} \tag{1.25}$$

This is an empirical result confirmed by laboratory experiments and practical experience. In Eq. (1.25), the internal energy U is a state variable (thermodynamic property). Hence, the change in internal energy dU is an exact differential and its value depends only on the initial and final states of the system. In contrast (the nonthermodynamic properties), δQ and δW depend on the process by which the system attained its final state from the initial state.

In general, for any given dU, there are an infinite number of ways (processes) by which heat can be added and work can be done on the system. In the present course of study, we will mainly be concerned with the following three types of processes only.

- *Adiabatic process:* A process in which no heat is added to or taken away from the system.
- *Reversible process:* A process which can be reversed without leaving any trace on the surroundings, that is both the system and the surroundings are returned to their initial states at the end of the reverse process.
- *Isentropic process:* A process which is adiabatic and reversible.

For an open system (e.g. pipe flow), there is always a term $(U + p\,\mathbf{V})$ instead of just U. This term is referred to as *enthalpy* or *heat function* H, and is given by

$$H = U + p\,\mathbf{V} \tag{1.26}$$

$$H_2 - H_1 = U_2 - U_1 + p_2\,\mathbf{V}_2 - p_1\,\mathbf{V}_1 \tag{1.27}$$

where $(p_2\mathbf{V}_2 - p_1\mathbf{V}_1)$ is termed *flow work*, and subscripts 1 and 2 represent states 1 and 2.

In general, we can say that the following are the major differences between the open and closed systems.

- The mass that enters or leaves an open system has kinetic energy, whereas there is no mass transfer possible across the boundaries of a closed system.
- The mass can enter and leave an open system at different levels of potential energy.
- Open systems are capable of delivering work continuously, because in the system the medium which transforms energy is continuously replaced. This useful work, which a machine continuously delivers, is called the *shaft work*.

1.9.1 Energy Equation for an Open System

Consider the system shown in Figure 1.8. The total energy E at the inlet station 1 and the outlet station 2 is given by

$$E_1 = U_1 + \frac{1}{2} m\, V_1^2 + m\,g\,z_1 \tag{1.28}$$

$$E_2 = U_2 + \frac{1}{2} m\, V_2^2 + m\,g\,z_2 \tag{1.29}$$

For an open system, the first-law expression given by Eq. (1.25) has to be rewritten with the total energy E in place of the internal energy U. Thus, we have

$$\boxed{Q_{12} - W_{12} = E_2 - E_1} \tag{1.30}$$

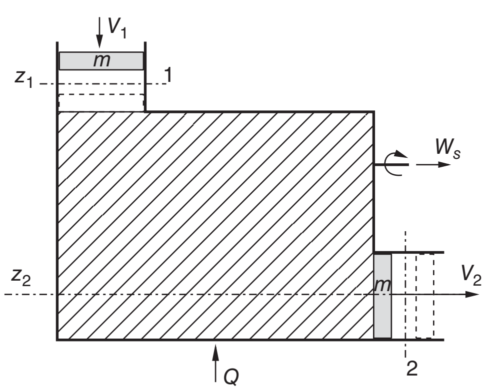


Figure 1.8 Open system.

Substituting for E_1 and E_2 from Eqs. (1.28) and (1.29), respectively, we get

$$Q_{12} - W_{12} = \left(U_2 + \frac{m}{2}V_2^2 + m\,g\,z_2\right) - \left(U_1 + \frac{m}{2}V_1^2 + m\,g\,z_1\right) \tag{1.31}$$

For an open system, the shaft (useful) work is not just equal to W_{12}, because the work done to move the pistons at 1 and 2 must also be considered. Work done with respect to the system by the piston at state 1 is

$$\begin{aligned} W_1' &= -F_1\Delta_1 \quad (F_1 = \text{force and } \Delta_1 = \text{displacement}) \\ W_1' &= -p_1 A_1 \Delta_1 \quad (p_1 = \text{pressure at 1}; A_1 = \text{cross-sectional area of piston}) \\ W_1' &= -p_1\mathbf{V}_1 \end{aligned}$$

Work delivered at 2 is $W_2' = p_2\mathbf{V}_2$. Therefore,

$$W_{12} = W_s + p_2\mathbf{V}_2 - p_1\mathbf{V}_1 \tag{1.32}$$

In Eq. (1.32), W_s is the shaft work, which can be extracted from the system, and $(p_2\mathbf{V}_2 - p_1\mathbf{V}_1)$ is the flow work necessary to maintain the flow. Substituting Eq. (1.32) into Eq. (1.31), we get

$$Q_{12} - W_s = \left(U_2 + p_2\mathbf{V}_2 + \frac{m}{2}V_2^2 + m\,g\,z_2\right) - \left(U_1 + p_1\mathbf{V}_1 + \frac{m}{2}V_1^2 + m\,g\,z_1\right)$$

or

$$Q_{12} - W_s = \left(H_2 + \frac{m}{2}V_2^2 + m\,g\,z_2\right) - \left(H_1 + \frac{m}{2}V_1^2 + m\,g\,z_1\right)$$

where $H_1 = U_1 + p_1\mathbf{V}_1$ and $H_2 = U_2 + p_2\mathbf{V}_2$. This is the fundamental equation for an open system. If there are any other forms of energy, such as electrical energy or magnetic energy, their initial and final values should be added properly to this equation. The energy equation for an open system

$$\boxed{H_1 + \frac{m}{2}V_1^2 + m\,g\,z_1 = H_2 + \frac{m}{2}V_2^2 + m\,g\,z_2 + W_s - Q_{12}} \tag{1.33}$$

is universally valid. This is the expression of the first law of thermodynamics for any open system. In most applications of gas dynamics, the gravitational energy is negligible compared to the kinetic energy. For working processes such as flow in turbines and compressors, the shaft work W_s in Eq. (1.33) is finite and for flow processes like flow around an airplane, $W_s = 0$. Therefore, for a gas dynamic working process, Eq. (1.33) becomes

$$H_1 + \frac{m}{2}V_1^2 = H_2 + \frac{m}{2}V_2^2 + W_s - Q_{12} \tag{1.34}$$

This is usually the case with systems such as turbo machines and internal combustion engines, where the process is assumed to be adiabatic (that is, $Q_{12} = 0$). For a gas dynamic adiabatic flow process, the energy Eq. (1.33) becomes

$$H_1 + \frac{m}{2}V_1^2 = H_2 + \frac{m}{2}V_2^2 \tag{1.35}$$

or

$$\boxed{H_1 + \frac{m}{2}V_1^2 = H_0 = \text{constant}} \tag{1.36}$$

where H_0 is the stagnation enthalpy and H_1 is the static enthalpy. That is, the sum of static enthalpy and kinetic energy is constant in an adiabatic flow.

1.9.2 Adiabatic Flow Process

For an adiabatic process, the heat transfer is associated with the process, $Q = 0$. Therefore, the energy equation is given by Eqs. (1.35) and (1.36). Dividing Eqs. (1.35) and (1.36) by m, we can rewrite them as

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{1.37}$$

$$h_1 + \frac{V_1^2}{2} = h_0 \tag{1.38}$$

or, in general,

$$\boxed{h + \frac{V^2}{2} = h_0 = \text{constant}} \tag{1.39}$$

where $h = H/m$ is called *specific static enthalpy* and h_0 is the specific stagnation enthalpy. With $h = p/\rho$, Eq. (1.39) represents Bernoulli's equation for incompressible flow,

$$p + \frac{1}{2}\rho\, V^2 = p_0 = \text{constant}$$

where p_0 is the stagnation pressure. That is, for the incompressible flow of air the energy equation happens to be the Bernoulli equation, because we are not interested in the internal energy and the temperature for such flows. In other words, Bernoulli's equation is the limiting case of the energy equation for incompressible flows. Here it is important to realize that, even though Bernoulli's equation for the incompressible flow of a gas is shown to be the limiting case of the energy equation, it is essentially a momentum equation. For a closed system,

$$Q_{12} - W_{12} = U_2 - U_1$$

In terms of specific quantities this becomes

$$q_{12} - w_{12} = u_2 - u_1$$

For the processes of a closed system there is no shaft work, that is no useful work can be extracted from the working medium. There will only be compression or expansion work. Therefore, w_{12} may be expressed as

$$w_{12} = \int_1^2 p\, dv$$

where v is the specific volume.
Thus, the change in internal energy becomes

$$du = \delta q - p\, dv \tag{1.40a}$$

Also, $h = u + pv$; $dh = du + p\ dv + v\ dp$. Using relation (1.40a), we can write the change in enthalpy as

$$dh = \delta q + v\, dp \tag{1.40b}$$

For adiabatic changes of state, Eqs. (1.40a) and (1.40b) reduce to

$$du = -p\, dv,\ \ dh = v\, dp \tag{1.40c}$$

where u, q, and v in Eqs. (1.40) stand for specific quantities of internal energy, heat energy, and volume, respectively.

1.10 The Second Law of Thermodynamics (Entropy Equation)

Consider a cold body coming into contact with a hot body. From experience, we can say that the cold body will get heated up and the hot body will cool down. However, Eq. (1.25) does not necessarily imply that this will happen. In fact, the first law allows the cold body to become cooler and the hot body to become hotter as long as energy is conserved during the process. However, in practice this does not happen; instead, the law of nature imposes another condition on the process, a condition that stipulates the direction in which a process should take place. To ascertain the proper direction of a process, let us define a new state variable, the entropy, as follows.

$$ds = \frac{\delta q_{\text{rev}}}{T} \tag{1.41}$$

where s is the entropy (amount of disorder) of the system, δq_{rev} is an incremental amount of heat added reversibly to the system, and T is the system temperature. The above definition gives the change in entropy in terms of a reversible addition of heat, δq_{rev}. Since entropy is a state variable, it can be used in conjunction with any type of process, reversible or irreversible. The quantity δq_{rev} is just an artifice; an effective value of δq_{rev} can always be assigned to relate the initial and final states of an irreversible process, where the actual amount of heat added is δq. Indeed, an alternative and probably more lucid relation is

$$\boxed{ds = \frac{\delta q}{T} + ds_{\text{irrev}}} \tag{1.42}$$

Equation (1.42) applies in general to all processes. It states that the change in entropy during any process is equal to the actual heat added, δq, divided by the temperature, $\delta q/T$, plus a contribution from the irreversible dissipative phenomena of viscosity, thermal conductivity, and mass diffusion occurring within the system, ds_{irrev}. These dissipative phenomena always cause an increase in of entropy:

$$ds_{\text{irrev}} \geq 0 \tag{1.43}$$

The equals sign in the inequality (1.43) denotes a reversible process where, by definition, the above dissipative phenomena are absent. Hence, a combination of Eqs. (1.42) and (1.43) yields

$$ds \geq \frac{\delta q}{T} \tag{1.44}$$

Further, if the process is adiabatic, $\delta q = 0$, and Eq. (1.44) reduces to

$$ds \geq 0 \tag{1.45}$$

Equations (1.44) and (1.45) are two forms of the second law of thermodynamics. The second law gives the direction in which a process will take place. Equations (1.44) and (1.45) imply that a process will always proceed in a direction such that the entropy of the system plus its surroundings always increases, or at least remains unchanged. That is, in an adiabatic process the entropy can never decrease. This aspect of the second law of thermodynamics is important because it distinguishes between reversible and irreversible processes.

If $ds > 0$, the process is called an *irreversible process*, and when $ds = 0$, the process is called a *reversible process*. A reversible and adiabatic process is called an *isentropic process*. However, in a nonadiabatic process, we can extract heat from the system and thus decrease the entropy of the system.

1.11 Thermal and Calorical Properties

The equation $pv = RT$ or $p/\rho = RT$ is called the *thermal equation of state*, where p, T, and $v(=1/\rho)$ are *thermal properties* and R is the gas constant. A gas that obeys the thermal equation of state is called a *thermally perfect gas*. Any relation between the calorical properties u, h, and s and any two thermal properties is called a *calorical equation of state*. In general, the thermodynamic properties (the properties do not depend on process) can be grouped into thermal properties (p, T, v) and calorical properties (u, h, s). From Eqs. (1.40), we have

$$u = u(T, v),\ \ h = h(T, p)$$

In terms of exact differentials, the above relations become

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \tag{1.46}$$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp \tag{1.47}$$

For a constant volume process, Eq. (1.46) reduces to

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT$$

where $\left(\frac{\partial u}{\partial T}\right)_v$ is the specific heat at constant volume represented as c_v; therefore,

$$du = c_v\, dT \tag{1.48}$$

For an isobaric process, Eq. (1.47) reduces to

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT$$

where $\left(\frac{\partial h}{\partial T}\right)_p$ is the specific heat at constant pressure represented by c_p; therefore,

$$dh = c_p\, dT \tag{1.49}$$

From Eq. (1.40a) for a constant volume (isochoric) process, we get

$$\delta q = du = c_v\, dT \tag{1.50a}$$

and for a constant pressure (isobaric) process,

$$\delta q = dh = c_p dT,\ \ \delta q = dh = c_v\, dT + p\, dv \tag{1.50b}$$

For an adiabatic flow process ($q = 0$), from Eq. (1.40c) we have

$$dh = v\, dp \tag{1.50c}$$

From Eqs. (1.50) it can be inferred that:

- If heat is added at constant volume, it only raises the internal energy.
- If heat is added at constant pressure, it not only increases the internal energy but also does some external work, that is it increases the enthalpy.
- If the change is adiabatic, the change in enthalpy is equal to the external work $v\, dp$.

1.11.1 Thermally Perfect Gas

A gas is said to be thermally perfect when its internal energy and enthalpy are functions of temperature alone, that is for a thermally perfect gas,

$$u = u(T),\ \ h = h(T) \tag{1.51a}$$

Therefore, from Eqs. (1.48) and (1.49), we get

$$c_v = c_v(T), \;\; c_p = c_p(T) \tag{1.51b}$$

Further, from Eqs. (1.46), (1.47), and (1.51a), we obtain

$$\left(\frac{\partial u}{\partial v}\right)_T = 0, \;\; \left(\frac{\partial h}{\partial p}\right)_T = 0 \tag{1.51c}$$

The important relations of this section are

$$\boxed{du = c_v\, dT, \;\; dh = c_p\, dT}$$

These equations are universally valid so long as the gas is thermally perfect. Otherwise, in order to have equations of universal validity, we must add $\left(\frac{\partial u}{\partial v}\right)_T dv$ to the first equation and $\left(\frac{\partial h}{\partial p}\right)_T dp$ to the second equation.

The state equation for a thermally perfect gas is

$$pv = RT$$

In the differential form, this equation becomes

$$p\, dv + v\, dp = R\, dT$$

Also,

$$h = u + pv$$
$$dh = du + p\, dv + v\, dp$$

Therefore,

$$dh - du = p\, dv + v\, dp = R\, dT$$

that is

$$R\, dT = c_p\, dT - c_v\, dT$$

Thus,

$$R = c_p(T) - c_v(T) \tag{1.52}$$

For thermally perfect gases, Eq. (1.52) shows that, though c_p and c_v are functions of temperature, their difference is a constant with reference to temperature.

1.12 The Perfect Gas

This is an even greater specialization than a thermally perfect gas. For a perfect gas, both c_p and c_v are constants and are independent of temperature, that is

$$c_v = \text{constant} \neq c_v(T), \;\; c_p = \text{constant} \neq c_p(T) \tag{1.53}$$

Such a gas, with constant c_p and c_v, is called a *calorically perfect gas*. Therefore, a perfect gas should be thermally as well as calorically perfect.

From the above discussions it is evident that:

- A perfect gas must be both thermally and calorically perfect.
- A perfect gas must satisfy both the *thermal equation of state:* $p = \rho\, R\, T$, and the *caloric equations of state:* $c_p = (\partial h/\partial T)_p$, $c_v = (\partial u/\partial T)_v$.

- A calorically perfect gas must be thermally perfect, but a thermally perfect gas need not be calorically perfect. That is, thermal perfectness is a prerequisite for caloric perfectness.
- For a thermally perfect gas, $c_p = c_p(T)$ and $c_v = c_v(T)$; that is, both c_p and c_v are functions of temperature. But even though the specific heats c_p and c_v vary with temperature, their ratio, γ, becomes a constant and is independent of temperature, that is γ = constant $\neq \gamma(T)$.
- For a calorically perfect gas, c_p and c_v, as well as γ are constants and independent of temperature.

1.12.1 Entropy Calculation

Entropy is defined by the relation (for a reversible process)

$$\delta q = T\,ds$$

Using Eq. (1.40), we can write

$$T\,ds = du + p\,dv \tag{1.54}$$

$$T\,ds = dh - v\,dp \tag{1.55}$$

Equations (1.54) and (1.55) are as important and useful as the original form of the first law of thermodynamics, Eq. (1.25).

For a thermally perfect gas, from Eq. (1.49), we have $dh = c_p\,dT$. Substituting this relation into Eq. (1.55), we obtain

$$ds = c_p \frac{dT}{T} - \frac{v\,dp}{T} \tag{1.56}$$

Substituting the perfect gas equation of state, $p\,v = R\,T$, into Eq. (1.56), we get

$$ds = c_p \frac{dT}{T} - R\frac{dp}{p} \tag{1.57}$$

Integrating Eq. (1.57) between states 1 and 2, we obtain

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R\,\ln\left(\frac{p_2}{p_1}\right) \tag{1.58}$$

Equation (1.58) holds for a thermally perfect gas. The integral can be evaluated if c_p is known as a function of T. Further, assuming the gas to be calorically perfect, for which c_p is constant, Eq. (1.58) reduces to

$$\boxed{s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\,\ln\left(\frac{p_2}{p_1}\right)} \tag{1.59}$$

Using $du = c_v\,dT$ in Eq. (1.54), the change in entropy can also be expressed as

$$\boxed{s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\,\ln\left(\frac{v_2}{v_1}\right)} \tag{1.60}$$

From the above discussion, we can summarize that a perfect gas is both thermally and calorically perfect. Further, a calorically perfect gas must also be thermally perfect, whereas a thermally perfect gas need not be calorically perfect.

For a thermally perfect gas, $p = \rho RT$, $c_v = c_v(T)$, $c_p = c_p(T)$, and for a perfect gas, $p = \rho RT$, c_v = constant and c_p = constant. Further, for a perfect gas, all equations get simplified, resulting in the following simple relations for u, h, and s.

$$u = u_1 + c_v T \tag{1.61a}$$

$$h = h_1 + c_p T \tag{1.61b}$$

$$s = s_1 + c_v \ln\left(\frac{p}{p_1}\right) - c_p \ln\left(\frac{\rho}{\rho_1}\right) \tag{1.61c}$$

where the subscript 1 refers to the initial state.

Equations (1.61a), (1.61b), and (1.52), combined with the thermal equation of state ($p = \rho RT$), result in

$$u = u_1 + \frac{1}{\gamma - 1}\frac{p}{\rho}, \quad h = h_1 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$$

where γ is the ratio of specific heats, c_p/c_v. For the simplest molecular model, the kinetic theory of gases gives the specific heats ratio γ as

$$\boxed{\gamma = \frac{n+2}{n}}$$

where n is the number of degrees of freedom of the gas molecules. Thus, for monatomic gases with $n = 3$ (only three translational degrees of freedom), the specific heats ratio becomes

$$\gamma = \frac{3+2}{3} = 1.67$$

Diatomic gases, such as oxygen and nitrogen, have $n = 5$ (three translational degrees of freedom and two rotational degrees of freedom), thus,

$$\gamma = \frac{5+2}{5} = 1.4$$

Gases with extremely complex molecules, such as freon and gaseous compounds of uranium, have large values of n, resulting in values of γ only slightly greater than unity. Thus, the value of specific heats ratio γ varies from 1 to 1.67, depending on the molecular nature of the gas:

$$\boxed{1 \leq \gamma \leq 1.67}$$

The above relations for u and h are important, because they connect the quantities used in thermodynamics with those used in gas dynamics. With the aid of these relations, the energy equation can be written in two different forms, as follows.

- The energy equation for an adiabatic process, as given by Eq. (1.39), is

$$h + \frac{V^2}{2} = h_0 = \text{constant}$$

When the gas is perfect, it becomes

$$c_p T + \frac{V^2}{2} = c_p T_0 = \text{constant} \tag{1.62a}$$

- Equation (1.62a), when combined with the state equation, yields

$$\boxed{\frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{V^2}{2} = \text{constant}} \tag{1.62b}$$

Equation (1.62b) is the form of energy equation commonly used in gas dynamics. This is popularly known as the *compressible Bernoulli's equation* for isentropic flows.

From Eq. (1.62a), we infer that for an adiabatic process of a perfect gas,

$$T_{01} = T_{02} = T_0 = \text{constant} \tag{1.63}$$

So far, we have not made any assumption about the reversibility or irreversibility of the process. Equation (1.63) implies that the stagnation temperature T_0 remains constant for an adiabatic process of a perfect gas, irrespective of the process being reversible or irreversible.

Consider the flow of gas in a tube with an orifice, as shown in Figure 1.9. In such a flow process, there will be pressure loss. But if the stagnation temperature is measured before and after the orifice plate and if it remains constant, then the gas can be treated as a perfect gas and all the simplified equations (Eqs. (1.61a)–(1.61c)) can be used. Otherwise, it cannot be treated as a perfect gas, and Eq. (1.61c) can be rewritten as

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \exp[(s_2 - s_1)/c_v] \tag{1.64}$$

1.12.2 Isentropic Relations

An adiabatic and reversible process is called an *isentropic process*. For an adiabatic process, $\delta q = 0$, and for a reversible process, $ds_{\text{irrev}} = 0$. Hence, from Eq. (1.42), an isentropic process is one for which $ds = 0$, that is the entropy is constant. Important relations for an isentropic process can be directly obtained from Eqs. (1.59), (1.60), and (1.64) by setting $s_2 = s_1$. For example, from Eq. (1.59) we have

$$0 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{c_p}{R}\ln\left(\frac{T_2}{T_1}\right)$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_p/R} \tag{1.65}$$

From Eq. (1.52),

$$c_p - c_v = R$$

$$1 - \frac{c_v}{c_p} = \frac{R}{c_p}$$

$$\frac{\gamma - 1}{\gamma} = \frac{R}{c_p}$$

since $c_p/c_v = \gamma$. Therefore,

$$\frac{c_p}{R} = \frac{\gamma}{\gamma - 1}$$

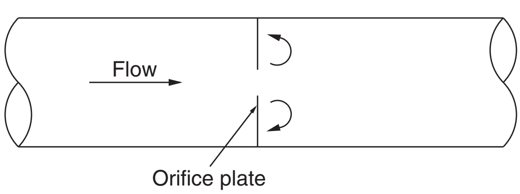


Figure 1.9 Flow through an orifice plate.

Substituting this relation into Eq. (1.65), we obtain

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \tag{1.66}$$

Similarly, from Eq. (1.60),

$$0 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{v_2}{v_1}\right) = -\frac{c_v}{R} \ln\left(\frac{T_2}{T_1}\right)$$

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{-c_v/R} \tag{1.67}$$

But it can be shown that

$$\frac{c_v}{R} = \frac{1}{\gamma - 1}$$

Substituting the above relation into Eq. (1.67), we get

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{-1/(\gamma-1)} \tag{1.68}$$

Since $\rho_2/\rho_1 = v_1/v_2$, Eq. (1.68) becomes

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \tag{1.69}$$

Substituting $s_1 = s_2$ into Eq. (1.64), we obtain

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{1.70}$$

This relation is also called *Poisson's equation*. Summarizing Eqs. (1.66), (1.69), and (1.70), we can write

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}} \tag{1.71}$$

Equation (1.71) is an important equation and is used very frequently in the analysis of compressible flows.

Using the isentropic relations discussed above, several useful equations of total (stagnation) conditions can be obtained as follows. From Eqs. (1.62a) and (1.15),

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{V^2}{2\gamma R\,T/(\gamma-1)} = 1 + \frac{V^2}{2a^2/(\gamma-1)}$$

where T is the static temperature, T_0 is the stagnation temperature and V is the flow velocity. Hence,

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2} \tag{1.72}$$

Equation (1.72) gives the ratio of total to static temperature at a point in an isentropic flow field as a function of the flow Mach number M at that point. Combining Eqs. (1.71) and (1.72), we get

$$\boxed{\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}} \tag{1.73}$$

$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}} \tag{1.74}$$

Equations (1.73) and (1.74) give the ratio of total to static pressure and total and static density, respectively, at a point in an isentropic flow field as a function of the flow Mach number M at that point. Equations (1.72)–(1.74) form a set of most important equations for total properties, which are often used in gas dynamic studies. Their values as a function of M for $\gamma = 1.4$ (air at standard conditions) are tabulated in Table A.1 of the Appendix.

At this stage we may ask how Eq. (1.71), which is derived from the concept of an isentropic change of state, which is so restrictive (adiabatic as well as reversible) that it may find only limited applications – is so important, and why it is frequently used. In compressible flow processes, such as flow through a rocket engine, flow over an airfoil, etc., large regions of the flow fields are isentropic. In the region adjacent to the rocket nozzle walls and the airfoil surface, a boundary layer is formed wherein the dissipative mechanisms of viscosity, thermal conduction, and diffusion are strong. Hence, the entropy increases within these boundary layers. On the other hand, for fluid elements outside the boundary layer, the dissipative effects are negligible. Further, no heat is being added to or removed from the fluid element at these points, hence the flow is adiabatic. Therefore, the fluid elements outside the boundary layer experience a reversible adiabatic process, hence the flow is isentropic. Moreover, the boundary layers are usually thin, hence large regime of flow fields are isentropic. Therefore, a study of isentropic flow is directly applicable to many types of practical flow problems. Equation (1.71) is a powerful relation, connecting pressure, density, and temperature, and is valid for calorically perfect gases.

Expressing all the quantities as stagnation quantities, Eq. (1.61c) can be written as

$$s_{02} - s_{01} = c_v \ln\left(\frac{p_{02}}{p_{01}}\right) - c_p \ln\left(\frac{\rho_{02}}{\rho_{01}}\right) \tag{1.75}$$

Also, from Eq. (1.52),

$$R = c_p - c_v$$

and from the state equation

$$\frac{p_{01}}{p_{02}} = \frac{\rho_{01}}{\rho_{02}} \frac{T_{01}}{T_{02}}$$

Substitution of the above relations into Eq. (1.75) yields

$$s_{02} - s_{01} = R \ln\left(\frac{p_{01}}{p_{02}}\right) + c_p \ln\left(\frac{T_{02}}{T_{01}}\right)$$

For an adiabatic process of a perfect gas,

$$T_{01} = T_{02}$$

(a)

(b)

Figure 12.73 Shadowgraph images of a Mach 2 circular jet at NPR 5, with sharp-edged tabs at 0.5D: (a) viewed normal to the tab; (b) viewed along the tab [96].

at the nozzle exit but also the combined effect of the pressure hill at the tab face and the level of expansion at the nozzle exit which dictates the size of the mixing-promoting small-scale vortices shed by the tab.

In the case of the uncontrolled elliptic nozzle, the size of the vortices shed at the nozzle exit would vary continuously from one end of the major axis to the adjacent end of the minor axis, in accordance with vortex theory, which states that the size of the vortex shed from an edge is proportional to the radius of curvature of the edge [97].

Thus, the azimuthal vortices shed around the ends of the major axis would be smaller than those shed around the ends of the minor axis. Schematic diagrams illustrating the azimuthal vortices shed from the exit of a circular and elliptic nozzle are shown in Figure 12.74.

It can be seen that the circular nozzle sheds only vortices of uniform size compared with the elliptic nozzle, which sheds vortices of a continuously varying size from one end of the major axis to the adjacent end of the minor axis. Because of this, the jet issuing from an elliptic nozzle will have a better mixing environment than a circular nozzle of identical area and inlet condition. This will enable the jet issuing from an elliptic nozzle to enjoy better aerodynamic mixing than an identical jet issuing from a circular nozzle.

When a tab is placed normal to a flow, it will shed vortices from its edges. These small-scale transverse vortices leaving the tab will become streamwise soon after shedding, owing to the inertia of the jet. The size of the vortex shed from a tab is proportional to its local half-width. The results of centerline pressure decay of the elliptic jet controlled with two identical rectangular and triangular tabs placed diametrically opposite, along the major and minor axes at the exit of nozzle, show that the tabs (rectangular and triangular) along the minor axis are more efficient

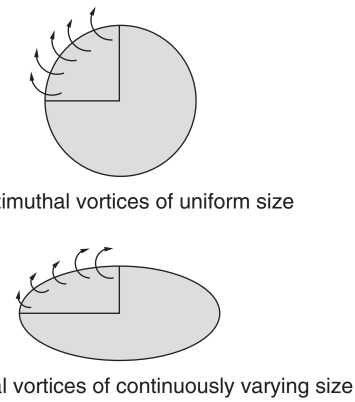


Figure 12.74 Schematic of vortices shed from (a) a circular nozzle exit and (b) an elliptic nozzle exit.

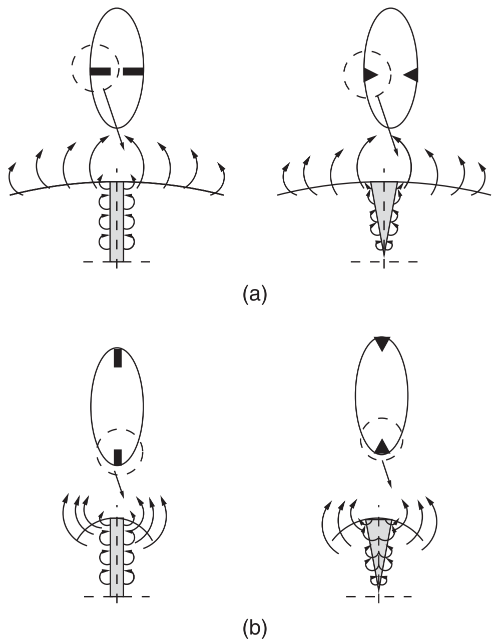


Figure 12.75 Schematic diagram of vortices shed from an elliptic nozzle exit and tabs: (a) rectangular and triangular tabs along the minor axis; (b) rectangular and triangular tabs along the major axis.

in enhancing the mixing of the jet with the ambient fluid than the tabs along the major axis [97]. This is because the azimuthal vortices shed at the minor axis side of the nozzle exit are relatively larger compared with the azimuthal vortices shed at the major axis side, owing to the curvature effect of the elliptic geometry. The relatively larger size of the azimuthal vortices shed around the minor axis, along with the relatively smaller small-scale vortices shed from the tabs placed along the minor axis (Figure 12.75a), seem to create a better mixing-promoting environment compared with those associated with the tabs along the major axis (Figure 12.75b). This might be the reason for the better mixing caused by the tabs along the minor axis. As can be seen in Figure 12.75, the width of the tab remains the same from base to tip for the rectangular tab compared with the triangular tab with varying width from base to vertex. Therefore, a triangular tab sheds vortices of continuously varying size from base to vertex, whereas a rectangular tab can shed only vortices of uniform size along its length [97].

Another important feature that dictates the size of the vortex shed from an object is the pressure hill at the front face [88]. For the triangular tab, the pressure hill at its face varies in size from its base to tip. Therefore, the environment required for mixing-promoting vortices of mixed size is built in for the triangular shape. The mixing promotion caused by triangular tabs along the minor axis would be better than that of rectangular tabs along the same orientation because of the mixed size of the mixing-promoting small-scale vortices shed by the triangular tab compared with the rectangular tab, which would shed small-scale vortices of uniform size only (Figure 12.75a). The small-scale vortices of continuously varying size from base to vertex, introduced by the triangular tabs, along with the small-scale azimuthal vortices of relatively larger size shed from the nozzle exit, would provide an environment of mixed-size vortices favorable for promoting mixing of the mass entrained by the large-scale vortices at the jet boundary with the jet mass.

The tabs in limit length are found to be effective in promoting the mixing. However, the studies on the mixing efficiency of limiting tabs are only few, especially on the use of limiting tabs for promoting the mixing of noncircular jets. The control effectiveness of a limiting tab on a supersonic elliptic jet issuing from an elliptic nozzle of AR 4 was studied by Rathakrishnan [98]. The dominant source for the generation of the mixing-promoting streamwise vortices shed by the tab is the pressure hill formed at the tab face of the tab. The size of the pressure hill is governed not only by the projected area of the tab normal to the flow but also by the shape of the tab. The pressure hills formed at the face of the flat, triangular, circular arc, and circular (crosswire) geometry of the same projected area will be different. The effect of this difference in the shape of the pressure hill is to make the tabs shed streamwise vortices of different sizes. The influence of this change in the size of mixing-promoting vortices shed by tabs on jet mixing was studied in the presence of various levels of pressure gradient, without going into the details of the actual size of the pressure hill.

Among the tabs studied, the triangular tab was found to be the most efficient mixing promoter for this Mach 2 elliptical jet. The triangular tab promotes mixing to the largest extent when placed along the minor axis, leading to a core length reduction of about 82%, in the presence of an adverse pressure gradient of about 36%, at the nozzle exit, that is at NPR 5. At this NPR, circular arc and flat tabs protect the core, that is these tabs retard the jet mixing, leading to an elongation of the core by 8.7 and 42.4% respectively, whereas the core reduction caused by crosswire is only 36.8% at this NPR. The triangular tab is also found to be the most efficient mixing promoter for tab orientation along the major axis, but for this orientation the best core length reduction caused is only 73.4%, at NPR 8. The efficient mixing promotion caused by the triangular tab does not introduce any abnormal level of asymmetry to jet propagation.

Core length variations caused by the limiting tabs of different geometries, placed along the major and minor axes of the AR 4 elliptic Mach 2 jet, at different levels of expansion, are quantified in Figures 12.76 and 12.77, respectively. The plots in Figure 12.76 show the variation of non-dimensional core length L/D with NPR, for tabs along the major axis. It can be seen that for an uncontrolled jet the core length increases monotonically with NPR, which is typical for a free jet. For controlled jets, the effect of NPR on the core length is less pronounced than for an uncontrolled jet. However, at the overexpanded state corresponding to NPR 4, establishing a high level of adverse pressure gradient at the nozzle exit, the circular arc tab is found to retard the mixing, but this retardation is only at NPR 4. For NPRs above 4, all the tabs enhance the mixing considerably, as can be seen from Figure 12.76.

Variation of the non-dimensional core length of the controlled and uncontrolled jets with NPR, for tab orientation along the minor axis, is shown in Figure 12.77. For this orientation of the tab also, at NPR 4, the circular arc tab is found to retard the mixing. Furthermore, the mixing caused by the flat tab is found to be insignificant at NPR 6, but the same flat tab is found to enhance the mixing to the highest level compared to other tabs for NPRs 7 and 8. Other tabs

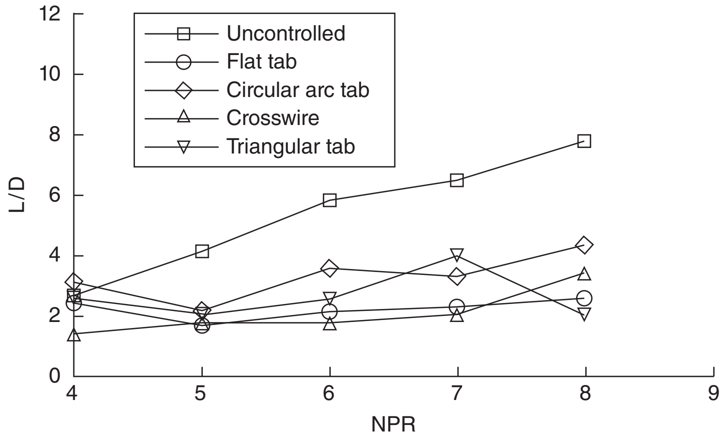


Figure 12.76 Jet core length variation with NPR, for tabs along the major axis [98].

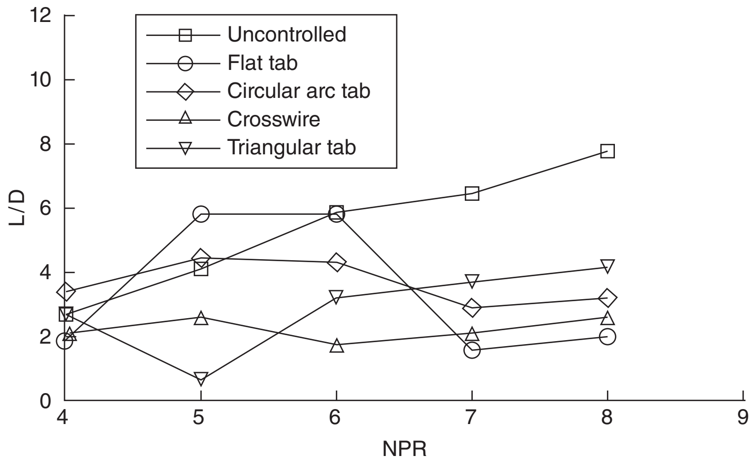


Figure 12.77 Core length variation with NPR for tabs along the minor axis [98].

also enhance the mixing considerably, but the mixing enhancement caused by the triangular tab at NPR 5 is the highest over the range of NPR, as well as across the tab geometries.

From these results it is evident that the mixing-promoting environment, established by the momentum-transporting small-scale vortices shed from the edges of the tabs and mass entraining large-scale vortices formed at the jet boundary, is strongly influenced by the combined effect of the pressure hill at the tab face, which is governed by the geometry of the tab, and the pressure gradient present at the nozzle exit, which governs the convection of mixing-promoting small-scale vortices shed by the tabs.

12.8 Summary

A jet may be defined as a *pressure-driven shear flow with the characteristic that the width-to-axial distance is a constant*. This constant assumes a value of 8 for jet Mach numbers < 0.2 and the constant decreases with an increase of Mach number.

When a jet is issuing into a still environment it is termed a *free jet* or a *submerged jet*. If it is issuing into a flow, constituting a field that has the jet surrounded by a flow field of different velocity, it is referred to as a *co-flowing jet*. When the jet is issuing normal to a boundary (either a solid or a fluid boundary), it is referred to as an *impinging jet*. If the jet axis is at an oblique angle to the boundary and the boundary is a wall, it is called a *wall jet*. If a jet is opposed by another jet, the combination is termed *opposing jets*.

For subsonic jets, the axial distance up to which the jet velocity along the axis is unaffected is called the *jet core*. The jet core is essentially a potential region where the jet retains its axial velocity at all axial points in the region. After the core the entire jet field is dominated by viscous action. The core for a subsonic free jet usually extends up to about six times the nozzle exit diameter (D). After that, the characteristic decay begins and that dominates from about $6D$ to about $12D$. The region from the end of characteristic decay to infinity is also referred to as a *self-similar region* or a *fully developed region*.

Large-scale vortices are efficient suction creators and thus efficient in engulfing the fluid from the surroundings into the jet. This process of bringing mass from the surroundings into the jet is termed *mass entrainment* or simply *entrainment*. The fluid mass with zero momentum from the surrounding zone, entrained by the large vortex structures, will interact with the jet fluid at a finite momentum. In the process the jet fluid will exchange momentum with the entrained mass. Thus, the momentum of the jet fluid will decrease and that of the entrained fluid will increase, but the total momentum of the jet is conserved. During the process, the large vortices get fragmented into small eddies. Small-scale eddies have a longer life and they also serve as good transporters of mass and momentum. Therefore, in a jet flow large- and small-scale structures must be present in suitable proportion to result in an efficient mixing of the jet flow with the surrounding fluid. However, identifying this proportion in a turbulent field such as a jet is an impossible task. Hence, it is usual practice to ensure the existence of these structures in appropriate proportion by indirect means. One of the popular means for this is the measurement of jet centerline decay, which is a direct measure of the mixing taking place in the jet field. A fast decay implies rapid mixing, and vice versa.

A jet with faster decay of its centerline pressure implies that it contains a proper combination of large and small vortices compared to an identical jet with a slower decay of centerline pressure. Enhancement of jet decay can be achieved with controls which are passive or active in nature. An *active control* is one which requires an additional source of energy for its action and a *passive control* is one that does not require any external energy for its action. Most of the passive control techniques make use of geometrical modification of the nozzle exit.

The velocity at the center of the section of an axially symmetric submerged jet is inversely proportional to the distance from the pole.

$$\boxed{u_m = \frac{\text{constant}}{x}}$$

The velocity decay along the axis of a plane-parallel jet is inversely proportional to the square root of the distance from the pole.

$$\boxed{u_m = \frac{\text{constant}}{\sqrt{x}}}$$

All free jets are turbulent, even at Reynolds numbers close to zero. This is because of the differential shear experienced by the jet at its periphery. Therefore, the laws governing the distribution of different pulsation characteristics of the stream and their interrelationships are very important in the theory of the turbulent jet in particular, and in the theory of turbulence in general.

Some of the well-known theories of the jet or the so-called free turbulence are

- Prandtl's old theory
- Taylor's theory
- Prandtl's new theory
- Richard's theory
- Mattioli's theory.

Basically, jets can be classified into incompressible jets and compressible jets. The jets with a Mach number of less than 0.3, up to which the compressibility effects are negligible, are called *incompressible jets*. Jets with a Mach number of more than 0.3 are termed *compressible jets*. Compressible jets can further be subdivided into subsonic, sonic, and supersonic jets. Jets with a Mach number of less than 1.0 are called *subsonic jets*, and jets with a Mach number of 1.0 are called *sonic jets*, which can be correctly expanded or underexpanded. Jets with a Mach number of more than 1.0 are called *supersonic jets*, and these can be further classified into *overexpanded*, *correctly expanded*, and *underexpanded jets*.

Subsonic jets are those with Mach numbers of between 0.3 and 1.0, and are always correctly expanded, and develop with an included angle of about 10°. The flow regimes in a subsonic jet are classified as follows.

- *Potential core region*. This region consists of a core zone of constant axial velocity equal to the jet (nozzle) exit velocity, surrounded by a rapidly growing and predominantly shear dominated annulus of mixing layer or shear layer with intense turbulence. The potential core of a subsonic jet typically extends to about five times the nozzle exit diameter (D_e) downstream of the nozzle exit. This is because the mixing initiated at the jet boundary (periphery) has not yet permeated into the entire flow field, thus leaving a region that is characterized by a constant axial velocity.
- *Transition region*. This is the region where the centerline velocity begins to decay. This characteristic decay zone extends from about $5D_e$ to $10D_e$ downstream, over which the turbulence changes from its annular to a somewhat pseudo-cylindrical distribution. As a result, the velocity difference between the ambient fluid and the high-speed core region of the jet decreases and attenuates the shear that supports the vortical rings in the jet, and thus the velocity profiles become smoother with jet propagation.
  The transition region is characterized by a growth of three-dimensional flow due to wave instability of the cores of the vortex rings. The merging of these distorted vortices produces large eddies which can remain coherent around the potential core region of the jet.
- *Fully developed region*. Beyond the transition region the jet becomes similar in appearance to a flow of fluid from a source of infinitely small thickness (in an axially symmetric case the source is a point, and in a plane parallel case it is a straight line perpendicular to the plane of flow of jet). In practice, the jet velocity becomes insignificant after about $30D_e$.

A jet is said to be overexpanded when the nozzle exit pressure p_e is lower than the ambient pressure p_a to which it is discharging.

A jet is said to be correctly expanded when the nozzle exit pressure is equal to the ambient pressure. This jet is also wave dominated, as is an imperfectly expanded jet, even though we think that there would not be any waves. The reason for this is that, as the jet is issuing from a confined area to an infinite area, it tries to expand through expansion waves and after that gets compressed through compression waves (the reflected waves from the jet boundary), and this results in a periodic wave structure.

A jet is said to be underexpanded when the nozzle exit pressure (p_e) is higher than the backpressure (p_b). Since the nozzle exit pressure is higher than the backpressure, wedge-shaped expansion waves occur at the edge of the nozzle. These waves cross one another and are reflected from the boundaries of the jet flow field as compression waves. The compression waves again cross one another and are reflected on the boundaries of the jet as expansion waves. For an underexpanded jet also, in addition to the expansion fan caused by the level of underexpansion, there will be another expansion effect, owing to the relaxation experienced by the jet on exiting from the nozzle into a large space. Thus, the combined effect of these two causes establishes a stronger expansion fan than the underexpansion level alone can establish. Beyond some limiting NPR, the shock intersecting point becomes an intersecting zone, which resembles a disc, as seen in Figure 12.9d for NPR 4.25. This disc is termed a *Mach disc*. The Mach disc is essentially a compression wave identical to a normal shock, across which the flow decelerates to a subsonic level. A sonic or supersonic jet with a Mach disc in the core is termed *highly underexpanded*.

The diverse nature of applicability jets demands that they be made suitable for a specific application, by controlling them. Here, control may be defined as the ability to modify the jet flow mixing characteristics to achieve engineering efficiency, technological ease, economy, adherence to standards, and so on.

Jet controls can be broadly classified into active and passive controls, but passive controls are the more desirable, partly because they require no external power source.

Large-scale coherent structures control the dynamics of all free shear flows, including plane mixing layers, jets of different geometries (axisymmetric, plane, elliptic, rectangular, and so on), and wakes. These two-dimensional structures were found to play an important role in entrainment and mixing processes in incompressible shear layers.

Mixing in supersonic shear layers is critically dependent upon the compressibility effects in addition to the velocity and density ratios across the shear layer. The compressibility level is best described by a parameter called the *convective Mach number*.

Supersonic jets normally possess complex shock patterns. Therefore, the role of shock waves in noise generation becomes significant. The main sources of high-speed noise are the turbulent nature of the flow, shock-turbulence interaction, flow-induced oscillations of shocks, and resonance effects. An intense, discrete acoustic emission termed *screech* or *whistle*, as a consequence of oscillating shock waves within a supersonic jet, usually dominates the noise emitted by a cold model converging jet operated at slightly above a choked flow condition.

The turbulent mixing noise is from both large-scale turbulence structures and fine-scale turbulence of the jet flow. The large-scale turbulence structures generate the dominant part of the turbulent mixing noise. The fine-scale turbulence is responsible for the background noise.

Broadband shock-associated noise and screech tones are generated only when a quasi-periodic shock cell structure is present in the jet core. The quasi-periodicity of the shock cells plays a crucial role in defining the characteristics of both the broadband and discrete frequency shock noises.

Noncircular jets, such as elliptic jets, owing to the continuous variation in the azimuthal radius of curvature, form flow structures of different size at the jet boundary and the jet spreads differently along different planes. Studies on noncircular jets show that the jet experiences fine-scale mixing at the corners and at high curvature regions and large suction at the low curvature/flat side regions, owing to the formation of large-scale vortices, resulting in large entrainment.

The increased entrainment and enhanced mixing capabilities of the elliptic jet (noncircular jets) relative to the equivalent circular jet are due to the phenomenon known as *axis-switching*, which is unanimously supported by the literature. Axis-switching is a phenomenon in which the cross-section of an asymmetric jet evolves in such a manner that, after a certain distance from the nozzle exit, the major and the minor axes are interchanged.

Waves (expansion/compression) present in the elliptic jet core are unsymmetrical and the waves in the circular jet core are symmetrical about the jet axis. This asymmetrical nature of the waves in the elliptic jet is due to the azimuthal asymmetry of the elliptic nozzle. Also, the shock cells in the core of the elliptic jet are shorter than the circular jet.

The distance from the orifice exit where axis-switching takes place depends on the orifice AR and the expansion level of the orifice. The increase of a favorable pressure gradient at the orifice exit results in the upward shifting of the axis-switching location, indicating an increase in near-field mixing.

The size of vortices shed from an object is proportional to the half-width of the object normal to the stream direction. For rectangular tab the half-width is uniform all along the tab length from the root end to the tip end, whereas the triangular tab, owing to its geometry, would shed vortices of continuously varying size all along its edges, with the largest at the root end and continuously decreasing in size toward the tip end. The isosceles triangular tab, capable of shedding vortices of continuously varying size along its edges, at every height from the base, would be of identical size, though of an opposite family. But, the mixing-promoting vortices shed from the right-angled triangular tab would be of a different size at all heights, in addition to being of an opposite family. Moreover, the vortices of continuously varying size shed from the opposite sides of the isosceles triangular tab would be inclined at an equal angle with respect to the axis of the tab. But the vortices from the opposite edges of the right-angled triangular tab are inclined with respect to the tab axis at different angles. Another important feature to be noted is that, near the sharp vertex tip, though the vortices shed would be of different size and opposite family, their closer proximity would make them interact intensely, leading to a loss of vorticity content. When the vertex is truncated, even at the tip, vortices of an opposite family would not interact among themselves. This might be an advantage because almost the entire vorticity content available with the mixing-promoting vortices would be used for mixing promotion.

Limiting tabs of triangular and circular (crosswire) cross-sectional geometry of a projected area of 5% of the nozzle exit area (that is a tab of 5% geometrical blockage), placed along the minor axis at the nozzle exit, were found to be efficient mixing promoters. The better mixing-promoting capability of the triangular tab compared to the crosswire may be because of the vertex of the triangle facing the flow, which might result in a reduced extent of pressure hill at the face. For a given tab geometry, the tab width can play a dominant role in modifying the mixing.

One of the serious shortcomings associated with the control of a jet with tabs is the momentum thrust loss caused by the blockage of the nozzle exit area by the tabs. Also, the detached shock envelope formed ahead of the tab will make the flow to deviate away from the nozzle centerline. Both the effects – the reduction of the nozzle exit area and the divergence of the jet – will lead to a loss of momentum thrust that the nozzle is capable of generating if the tabs are not at the nozzle exit. Therefore, it will be of great value, from both the propulsion and application points of view, if the aerodynamic mixing of the jet with the surrounding environment to which it is discharged is enhanced using a control, which will augment the jet mixing without any adverse effect (such as a decrease of jet velocity). This concept of jet control with the tab positioned slightly downstream of the nozzle exit is termed *shifted tabs*.

Mixing modification caused by limiting rectangular tab with and without corrugations located at the nozzle exit and at 0.5D, downstream of the nozzle exit, for subsonic and sonic jet was found to be inferior to a tab at the nozzle exit.

For the case of a Mach 2 jet, mixing promotion caused by a shifted tab was found to increase with an increase of adverse pressure gradient. In fact, the mixing enhancement caused by a tab placed at the nozzle exit decreases with an increase of the adverse pressure gradient.

The efficacy of passive control in the form of a ventilated triangular tab on the mixing characteristics of supersonic jets to apprehend the governing flow physics and to capture the flow structure, in the presence of different levels of expansion at the exit of a Mach 1.5 elliptic nozzle of AR 3, revealed that an important geometrical feature, which influenced the mixing modification caused by both ventilated and unventilated triangular tab, is its vertex angle.

Study of the effect of modifying the shape of tab edges, on the aerodynamic mixing characteristics of a circular Mach 2.0 jet, controlled with a rectangular tab, revealed that when placed at the same location from the nozzle exit a sharp-edged tab is a better mixing promoter than a square-edged tab, for a highly overexpanded jet. At a marginal level of a positive pressure gradient, the square-edged tab is found to be a far superior mixing promoter than the sharp-edged tab, in all the three zones of the jet. This mixing-promoting superiority of the square-edged tab increases with a decrease of the adverse pressure gradient and an increase of a favorable pressure gradient.

An important feature that dictates the size of the vortex shed from an object is the pressure hill at the front face. For the triangular tab, the pressure hill at its face varies in size from its base to its tip. Therefore, the environment required for mixing-promoting vortices of mixed size is built in for the triangular shape. The mixing promotion caused by triangular tabs along the minor axis would be better than that of rectangular tabs along the same orientation because of the mixed size of the mixing-promoting small-scale vortices shed by the triangular tab compared with the rectangular tab, which would shed small-scale vortices of uniform size only. The small-scale vortices of continuously varying size from base to vertex, introduced by the triangular tabs, along with the small-scale azimuthal vortices of a relatively larger size shed from the nozzle exit, would provide an environment of mixed-size vortices, which is favorable for promoting mixing of the mass entrained by the large-scale vortices at the jet boundary with the jet mass.

Appendix A

Table A.1 Isentropic flow of perfect gas ($\gamma = 1.4$).

M	p/p_0	T/T_0	ρ/ρ_0	A/A^*	a/a_0	M^*	μ	ν
0.00	1.0000	1.0000	1.0000	∞	1.0000	0.0000		
0.01	0.9999	1.0000	1.0000	57.874	1.0000	0.0110		
0.02	0.9997	0.9999	0.9998	28.942	1.0000	0.0219		
0.03	0.9994	0.9998	0.9996	19.301	0.9999	0.0329		
0.04	0.9989	0.9997	0.9992	14.481	0.9998	0.0438		
0.05	0.9983	0.9995	0.9988	11.591	0.9998	0.0548		
0.06	0.9975	0.9993	0.9982	9.666	0.9996	0.0657		
0.07	0.9966	0.9990	0.9976	8.292	0.9995	0.0766		
0.08	0.9955	0.9987	0.9968	7.262	0.9994	0.0876		
0.09	0.9944	0.9984	0.9960	6.461	0.9992	0.0985		
0.10	0.9930	0.9980	0.9950	5.822	0.9990	0.1094		
0.11	0.9916	0.9976	0.9940	5.299	0.9988	0.1204		
0.12	0.9900	0.9971	0.9928	4.864	0.9986	0.1313		
0.13	0.9883	0.9966	0.9916	4.497	0.9983	0.1422		
0.14	0.9864	0.9961	0.9903	4.182	0.9980	0.1531		
0.15	0.9844	0.9955	0.9888	3.910	0.9978	0.1639		
0.16	0.9823	0.9949	0.9873	3.673	0.9974	0.1748		
0.17	0.9800	0.9943	0.9857	3.464	0.9971	0.1857		
0.18	0.9776	0.9936	0.9840	3.278	0.9968	0.1965		
0.19	0.9751	0.9928	0.9822	3.112	0.9964	0.2074		
0.20	0.9725	0.9921	0.9803	2.964	0.9960	0.2182		
0.21	0.9697	0.9913	0.9783	2.829	0.9956	0.2290		
0.22	0.9668	0.9904	0.9762	2.708	0.9952	0.2398		
0.23	0.9638	0.9895	0.9740	2.597	0.9948	0.2506		
0.24	0.9607	0.9886	0.9718	2.496	0.9943	0.2614		
0.25	0.9575	0.9877	0.9694	2.403	0.9938	0.2722		
0.26	0.9541	0.9867	0.9670	2.317	0.9933	0.2829		
0.27	0.9506	0.9856	0.9645	2.238	0.9928	0.2936		
0.28	0.9470	0.9846	0.9619	2.166	0.9923	0.3043		

(*continued*)

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Table A.1 (Continued)

M	p/p_0	T/T_0	ρ/ρ_0	A/A^*	a/a_0	M^*	μ	ν
0.29	0.9433	0.9835	0.9592	2.098	0.9917	0.3150		
0.30	0.9395	0.9823	0.9564	2.035	0.9911	0.3257		
0.31	0.9355	0.9811	0.9535	1.977	0.9905	0.3364		
0.32	0.9315	0.9799	0.9506	1.922	0.9899	0.3470		
0.33	0.9274	0.9787	0.9476	1.871	0.9893	0.3576		
0.34	0.9231	0.9774	0.9445	1.823	0.9886	0.3682		
0.35	0.9188	0.9761	0.9413	1.778	0.9880	0.3788		
0.36	0.9143	0.9747	0.9380	1.736	0.9873	0.3893		
0.37	0.9098	0.9733	0.9347	1.696	0.9866	0.3999		
0.38	0.9052	0.9719	0.9313	1.659	0.9859	0.4104		
0.39	0.9004	0.9705	0.9278	1.623	0.9851	0.4209		
0.40	0.8956	0.9690	0.9243	1.590	0.9844	0.4313		
0.41	0.8907	0.9675	0.9207	1.559	0.9836	0.4418		
0.42	0.8857	0.9659	0.9170	1.529	0.9828	0.4522		
0.43	0.8807	0.9643	0.9132	1.501	0.9820	0.4626		
0.44	0.8755	0.9627	0.9094	1.474	0.9812	0.4729		
0.45	0.8703	0.9611	0.9055	1.449	0.9803	0.4833		
0.46	0.8650	0.9594	0.9016	1.425	0.9795	0.4936		
0.47	0.8596	0.9577	0.8976	1.402	0.9786	0.5038		
0.48	0.8541	0.9559	0.8935	1.380	0.9777	0.5141		
0.49	0.8486	0.9542	0.8894	1.359	0.9768	0.5243		
0.50	0.8430	0.9524	0.8852	1.340	0.9759	0.5345		
0.51	0.8374	0.9506	0.8809	1.321	0.9750	0.5447		
0.52	0.8317	0.9487	0.8766	1.303	0.9740	0.5548		
0.53	0.8259	0.9468	0.8723	1.286	0.9730	0.5649		
0.54	0.8201	0.9449	0.8679	1.270	0.9721	0.5750		
0.55	0.8142	0.9430	0.8634	1.255	0.9711	0.5851		
0.56	0.8082	0.9410	0.8589	1.240	0.9700	0.5951		
0.57	0.8022	0.9390	0.8544	1.226	0.9690	0.6051		
0.58	0.7962	0.9370	0.8498	1.213	0.9680	0.6150		
0.59	0.7901	0.9349	0.8451	1.200	0.9669	0.6249		
0.60	0.7840	0.9328	0.8405	1.188	0.9658	0.6348		
0.61	0.7778	0.9307	0.8357	1.177	0.9647	0.6447		
0.62	0.7716	0.9286	0.8310	1.166	0.9636	0.6545		
0.63	0.7654	0.9265	0.8262	1.155	0.9625	0.6643		
0.64	0.7591	0.9243	0.8213	1.145	0.9614	0.6740		
0.65	0.7528	0.9221	0.8164	1.136	0.9603	0.6837		
0.66	0.7465	0.9199	0.8115	1.127	0.9591	0.6934		
0.67	0.7401	0.9176	0.8066	1.118	0.9579	0.7031		
0.68	0.7338	0.9153	0.8016	1.110	0.9567	0.7127		

(*continued*)

dial-type pressure gauge 332
diffuser 75
  contraction ratio 79, 362
  efficiency 84
    relation for 85
  isentropic efficiency of 358, 359
  normal shock 77, 215
  oblique shock 77, 215
  polytropic efficiency of 358
  supersonic 76, 78, 412
    special features of 77
  of supersonic tunnel 78, 358
discharge coefficient of nozzle 73
discharge from reservoir 45
discontinuity surface 180
dissimilar metals 335
disturbance waves 10, 155
  propagation of 9, 10
double wedge 199, 427
drag coefficient 203, 206, 242, 252
  due to camber 206
  due to lift 206, 385
  due to thickness 206
  intake 415, 432
  per unit span 198
  wave drag 198, 211, 261
dynamic pressure 4, 88, 203
  compressibility correction for 91
  correction coefficient 92

e

effects of second throat 360
efficiency
  aerodynamic 261
  of diffuser 84
    isentropic 358, 410
    polytropic 358
  of nozzle 71
elliptic equation 229, 239, 253, 271, 310
elliptic jet 473, 519, 523
  shadowgraph image of 522, 529
energy equation 1, 7, 11, 19, 115
  for an adiabatic flow 14
  for isentropic flow process 45
  for an open system 12, 33
engine inlets 404
  subsonic inlets 404
  supersonic inlets 411
enthalpy 7, 12, 223
  specific 14
  stagnation 13
  static 13
entrainment 451, 453
entropy 11
  calculation 18
  definition of 15
  discontinuity 180
  equation 15
equation of state
  calorical 16
  for perfect or ideal gas 3
  thermal 16
Euler's equation 171, 225, 235
expansion
  centered 67, 138
  continuous 67
  corner 156
  fan 66, 156
    reflection of 184
    strength of 139
  Prandtl-Meyer 170
  wave 1, 61
  work 14

f

Fanno
  curves 284
  flow 283
  line 284
  table of 604
first law of thermodynamics 11
  for an open system 13
fixed-geometry inlets 83
flow
  adiabatic 14
  deflection angle 155, 165
  with friction 283
  with heating or cooling 294
  inviscid 44
  isentropic 2
  mass rate of 2, 44, 47
  one-dimensional 43
  potential 228 *see also* potential flow
  two-dimensional 27, 155, 164, 169
flow through inlets 405
flow visualization 192, 339, 390
force measurement 389
free molecular flow 130

friction coefficient
  definition of 286
fringe-displacement method 344

g
gas
  calorically perfect 17
  calorical properties 16
    perfect 6, 17
  thermally perfect 16
  thermal properties 16
gas constant 7, 16
  universal 49
gas dynamics 1
  definition of 1
general region 319
Gladstone-Dale constant 341
Gladstone-Dale equation 341
Gothert's rule 240, 244, 247, 252, 254
  for finite wings 260
  for subsonic flow 244
  for supersonic flow 247
  three-dimensional flow 252
  two-dimensional flow 244

h
heating value (HV) definition of 399
higher heating value (HHV) 399
high-speed
  flow 11, 139, 283, 335
  tunnels 354
hodograph plane 161
hot-wire anemometer 383
  constant current type (CCA) 391
  constant temperature
    type (CTA) 391
Hugoniot
  curve 122
  equation 121, 122, 147
hyperbolic equation 255, 311
hypersonic 31
  flow 31, 64, 168, 228, 236, 250
  nozzle 377
  similarity 250
  similarity parameter 251, 269
  tunnel 354, 375, 376

i
ideal gas 3, 26
ideal ramjet 396
incompressible
  Bernoulli's equation 3, 14
  flow 1, 2, 11, 26
intake
  center body 424
  Concorde 431
  double-wedge 428
  inlet flow process 406
  multi-shock 427
  pitot 415
  single-wedge 424
  two-shock 423
interferometer 340, 341
introduction to free jets 451
irreversible processes 33
irrotational flow 224
isenthalpic flow 1
isentropic compression 24, 167, 429
  waves 165, 167
isentropic efficiency 358, 359, 410
isentropic flow 2
  table 547
isentropic index 6
isentropic process 12
  definition of 15
  equation for 6
isentropic relations 20

j
jet 451
  acoustic characteristics 490, 496
  centerline decay 451, 453, 461, 468
  characteristic decay 452
  classification of 454
  co-flowing 451
  compressible 453
  control of 453
    active control 473
    with the limiting tab 481
    passive control 473
  control with tabs 471
  core 452
  correctly expanded 61, 467
    centerline decay 63
    waves in 62
  definition of 451
  elliptic 519
  entrainment 451

expansion levels of 464
flow development 477, 480
free 451
turbulence characteristics 457
fully developed zone 452, 454
impinging 451
incompressible 453
noise 462
amplitude 463, 493
frequency 491
mixing 462
screech 462, 480
shock associated 462
noncircular 519
opposing 451
overexpanded 453, 454, 464–467
centerline decay 466
potential core 454
pressure profiles 504
shadowgraph picture 470
sonic 453
submerged 451
subsonic 453
supersonic 453
theory of 455, 456
transition region 454
turbulence characteristics of 457
underexpanded 469
centerline decay 472
shadowgraph picture 470
water flow visualization 517

k

King's law 338
knife-edge 344, 348
Knudsen number 130

l

Laplace's equation 6, 224
basic solutions of 224
lift 198, 202
coefficient of 203, 205
light source 341, 348
like reflection 183, 185, 192, 211
limits of external compression 431
linearized potential flow equation 228, 236
liquid manometers 329
measuring principle 330
losses in supersonic tunnels 357
expansion loss 358
frictional losses 358
losses in contraction cone 358
losses in cooling system 358
losses in guide vanes 358
losses in test-section 358
losses due to model and support system drag 358
losses due to shock wave 358
lower heating value (LHV) 399

m

Mach angle 8, 157
Mach cone 10, 27
Mach number relation 163
Mach disc 342, 353, 469, 471, 536
Mach line 9, 32, 163, 171
Mach number 4, 28
characteristic 53
critical 243, 256, 261
definition of 4, 5
determination 381, 388
relation to area 58
relation to density 22
relation to pressure 22
relation to temperature 21
relation to total pressure 76
of shock wave 126
Mach reflection 191
Mach wave 9, 27, 32, 116, 157, 189
manometers 329
measuring principle of 330
multi-tube 330
U-tube 330
mass flow rate 2
maximum 65
per unit area 47
relation 65
mass-motion velocity 126
material boundary 180
maximum velocity 46, 53, 406
measurements in compressible flow 329
density problems 339
pressure measurements 329
temperature measurements 335
velocity and direction 338
method of characteristics 309
axisymmetric flow 316
boundary points 313

method of characteristics (*contd.*)
compatibility relations 310
computational method 312
concepts of characteristics 309
design of supersonic nozzle 323
nonisentropic flow 317
numerical computation 312, 320
sources of error 316
theorem for 2-D flow 318
mixing length 457, 458
momentum equations 1, 171, 235
momentum thrust 74
loss 479, 533
relation of 527
most amplified frequency 473
moving shock 123
density ratio across 126
governing equations 124
Mach number 126
temperature ratio across 125
velocity 126

n

natural coordinates 221, 311
noncircular jets 473, 519, 544
nonsimple region 178, 211
normal shock 113
density ratio across 116
diffuser 76
in ducts 420
entropy change across 119, 146
equation for 113
flow through 114
Hugoniot equation 121
Prandtl relation of 116
pressure ratio across 117
reflected 133
Mach number 135
relations 115
speed of 123, 126
stagnation pressure ratio across 76, 119
strength of 117
strong 130
table 559
thickness of 113, 148
wave 113
weak 128
nozzle 57
contoured 67
flow process 69
convergent 65, 68, 70, 97
convergent–divergent 60
correctly expanded 61
de Laval 57
design 323
discharge coefficient 73
efficiency 71
flow process 69
isentropic expansion 60, 66
overexpanded 61, 70, 98
pressure ratio (NPR) 69
sharp cornered 325
straight 68
supersonic 164, 323
underexpanded 61
Nusselt number 338

o

oblique shock 155
chart 1 616
chart 2 617
flow through 156
intersection of 178, 180
limit as Mach wave 157, 159, 163
maximum deflection for 157, 159
reflection of 178
relations 156
solution 159
strong 160
table 569
transformed from normal shock 156
wave 155
weak 160
one-dimensional approximation 43
one-dimensional flow 43
with area change 54
with friction 283
with heating 294
open system 33
energy equation 13
optical methods 339
overall efficiency 401
overall sound pressure level (OASPL) 491

p

passive control 473

perfect gas 33
adiabatic flow 14
calorically perfect gas 17, 18
definition of 17
entropy of 18
isentropic flow of 2
limitations of 25
normal shock in 114
specific heats of 6, 7, 18
speed of sound in 27
performance of actual intakes 410
perturbation
potential 228, 236, 277, 280
velocities 227, 230, 233
P–G rule 242, 243, 245–248, 252, 254, 258–261, 267, 268
piston problem 123
pitot
intakes 415
pressure 88, 382
measurement 382
tube 88, 382, 481
pitot-static tube 91
plane waves 123
polytropic efficiency 358
potential equation
for bodies of revolution 229
linearization of 226
supersonic flow 278
for 3-D flow 225
potential flow 28, 221, 228, 230, 235, 236, 253, 431
potential function 224
for doublet 225
for free vortex 225
for source 225
for uniform flow 224
Prandtl–Glauert (P-G) rule
for subsonic flow 239
for supersonic flow 245
Prandtl–Meyer expansion 170
Prandtl–Meyer flow 171
Prandtl–Meyer function 175
Prandtl number 28, 336
Prandtl relation 116
pressure
dynamic 4, 88, 204
geometric 88
hill 478, 534
pitot 88, 381
indicated 89
static 88
measurement of 89, 382
total 88
transducers 329, 333
pressure coefficient 95, 203
for incompressible flows 97
pressure ratio, critical 52, 98
process
adiabatic 12
constant pressure 33
constant volume 33
irreversible 12, 15, 33
isentropic 12, 15, 20
reversible 15, 20, 33
propagation of disturbance waves 8, 9, 27
propulsive efficiency 400, 401

q

quasi-one-dimensional flow 43, 54

r

ramjet 395
engine 396
ideal 396
Rathakrishnan limit 479
Rayleigh flow 283
equations 294
Rayleigh line 294
relation 302
Rayleigh supersonic pitot formula 89, 99
recovery factor 336, 337
reflection
of an expansion fan 184
from free boundary 185, 321
like 185, 192
unlike 185, 465
Mach 191
of oblique shock 184
unlike 185, 211, 465
of waves 321
reservoir
discharge from 45
reversible process 12
revolution
body of 229, 232, 234
boundary condition 233

revolution (*contd.*)
  coefficient of pressure 237
Reynolds number 28
  control 370
  effects 385
rotation 156, 223

S
Schlieren method 339, 344, 350
  sensitivity of 350
scramjet 411, 442, 446, 471
second throat 76, 361
  effects of 360
Seebeck effect 335
  principle 335
self-similar
  flow 452
  motion 211
  region 452, 542
shadowgraph 352, 391
shaft work 12, 13
sharp cornered nozzle 325
shear layer 348, 451, 454, 473
  role in flow control 474
  subsonic 474
  supersonic 475
shear stress 29, 285–288
shifted tabs 519, 527, 530–536
shock 9, 27
  angle 10
  barrel 66
  definition of 113
  detached 119, 160, 189–190
  Mach number 126
  moving 123
  normal 113
    equations of motion 113
  oblique 70
  reflected 133
  speed 128
  strength 117
  strong 130
  velocity 126
  weak 128
shock cell 66
shock expansion theory 197
shock polar 160, 211
  relation 162
shock-rhombus 357
shock swallowing 362, 416
shock tube 139
  applications 142
  diaphragm pressure ratio 140
  driver section of 147
  equation 141, 147
  expansion section 147
  flow motion in 140
short duration light source 348
silence zone 9
similarity parameters 28
similarity rule 239, 245, 252 *see also* P-G rule; Gothert's rule
simple region 178, 211
simple T_0 change 294
  relations for a perfect gas 295
simple waves 178, 211
slender bodies 226
slipstream 180, 184, 191
slip surface 180
small perturbation theory 227, 236
sonic velocity 10, 59, 97
specific enthalpy 14
specific heats 7, 18
  ratio 6, 19
  ratio for thermally perfect gas 18
  ratio value range of 19
specific thrust 400
specific volume 14
speed of sound 5
  expression 7
stagnation enthalpy 14
stagnation pressure 14 *see also* total pressure
  stagnation pressure ratio 411
stagnation temperature 21 *see also* total temperature
stagnation temperature probe 337
starting load 355, 356, 385
starting problem 413
state equation
  calorical state equation 17
  for perfect gas 7
  for a thermally perfect gas 16, 17
  thermal state equation 17
static pressure definition of 88
  measurement 382
  probe 90
streamline 43
streamtube 43